Classical integrable structure of Schrodinger sigma model

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1. Introduction and Conclusion

Our aim : Integrable deformations of AdS/CFT

Consider the **Schrodinger spacetimes**.

- 1-parameter deformation of AdS spaces
- gravity duals for non-relativistic CFTs

Consider the 3D Schrodinger spacetime:

$$ds^2 = L^2\left[d\rho^2 - 2\mathrm{e}^{-2\rho}dudv - C\mathrm{e}^{-4\rho}dv^2\right]$$

Isometry : SL(2,R)$_L$ x U(1)$_R$

With $g = \mathrm{e}^{2vT^+}\mathrm{e}^{2\rho T^2}\mathrm{e}^{2uT^-} \in SL(2,\mathbb{R})$

$$ds^2 = \frac{L^2}{2}\left[\mathrm{tr}\left(J^2\right) - 2C\left(\mathrm{tr}\left[T^- J\right]\right)^2\right] \qquad J = g^{-1}dg$$

SL(2,R)$_L$ x U(1)$_R$ symmetry : $g' = g_L \cdot g \cdot \mathrm{e}^{-T^-\delta u}$

Non-linear sigma model defined on 3D Schrodinger spacetime (Schrodinger sigma model)

Action

$$S = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dx\ \eta^{\mu\nu}\left[\mathrm{tr}\left(J_\mu J_\nu\right) - 2C\mathrm{tr}\left(T^- J_\mu\right)\mathrm{tr}\left(T^- J_\nu\right)\right]$$

Boundary conditions : $g(t,x) \to g_\infty\ (=\mathrm{const.})\,,\quad \text{for } x \to \pm\infty$

Hybrid integrable structure of Schrodinger sigma model

Global symm.	SL(2,R)$_L$	U(1)$_R$
Hidden symm.	Yangian	????
r-matrix	rational	deformed rational

2. Left description [I.K., K.Yoshida, 1109.0872]

SL(2,R)$_L$ current :

$$j^L_\mu = gJ_\mu g^{-1} - 2C\mathrm{tr}(T^- J_\mu)gT^- g^{-1} - \underbrace{\sqrt{C}\epsilon_{\mu\nu}\partial^\nu\left(gT^- g^{-1}\right)}_{\text{topological term}}$$

The SL(2,R)$_L$ current satisfies the flatness condition: $\left[\partial_t - j^L_t, \partial_x - j^L_x\right] = 0$

FACT

flat and conserved current

→ infinite number of conserved charges

$$Q^L_{(0)} = \int_{-\infty}^{\infty} dx\ j^L_t(x)$$

cf. BIZZ construction

$$Q^L_{(1)} = \frac{1}{4}\int_{-\infty}^{\infty} dx\int_{-\infty}^{\infty} dy\ \epsilon(x-y)\left[j^L_t(x), j^L_t(y)\right] - \int_{-\infty}^{\infty} dx\ j^L_x(x)$$

$$Q^L_{(2)}\,,\ Q^L_{(3)}\,, \cdots$$

→ They satisfy the defining relations of Yangian.

Left Lax pair, classical *r*-matrix

$$L^L_t(\lambda_L) = \frac{1}{1-\lambda_L^2}\left[j^L_t - \lambda_L j^L_x\right]\,,\quad L^L_x(\lambda_L) = \frac{1}{1-\lambda_L^2}\left[j^L_x - \lambda_L j^L_t\right]$$

λ_L : spectral parameter

→ $\left[\partial_t - L^L_t(\lambda_L), \partial_x - L^L_x(\lambda_L)\right] = 0$

Poisson brackets of $L^L_x(\lambda_L)$ → rational *r*-matrix

Monodromy matrix

$$U^L(\lambda_L) = \mathrm{P}\exp\left[\int_{-\infty}^{\infty} dx\ L^L_x(x;\lambda_L)\right] \to \frac{d}{dt}U^L(\lambda_L) = 0$$

Infinite dim. symm. can be obtained by expanding the monodromy matrix.

EX. Yangian algebra is obtained from $U^L(\lambda_L)$.

3. Right description [I.K., K.Yoshida, 1109.0872; I.K., T.Matsumoto, K.Yoshida, in progress]

U(1)$_R$ symm. Is enhanced with non-local currents:

$$\begin{cases} j^{R,-}_\mu = -J^-_\mu \\ j^{R,2}_\mu = -\mathrm{e}^{\sqrt{C}\chi}\left(J^2_\mu + \sqrt{C}\epsilon_{\mu\nu}J^{\nu,-}\right) \\ j^{R,+}_\mu = -\mathrm{e}^{\sqrt{C}\chi}\left(J^+_\mu + \sqrt{C}\epsilon_{\mu\nu}J^{\nu,2} + CJ^-_\mu\right) \end{cases}$$

$$\chi(x) = -\frac{1}{2}\int_{-\infty}^{\infty} dy\ \epsilon(x-y)j^{R,-}_t(y)$$

$$\partial^\mu\chi = -\epsilon^{\mu\nu}j^{R,-}_\nu$$

→ $Q^{R,-} = \int_{-\infty}^{\infty} dx\ j^{R,-}_t(x)\,,\ Q^{R,2} = \int_{-\infty}^{\infty} dx\ j^{R,2}_t(x)\,,\ Q^{R,+} = \int_{-\infty}^{\infty} dx\ j^{R,+}_t(x)$

Algebra of non-local charges (*q*-deformed Poincare symm.)

$$\left\{Q^{R,-}, Q^{R,2}\right\}_\mathrm{P} = \frac{2}{\sqrt{C}}\sinh\left(\frac{\sqrt{C}}{2}Q^{R,-}\right)$$

$$\left\{Q^{R,-}, Q^{R,+}\right\}_\mathrm{P} = Q^{R,2}\,,\quad \left\{Q^{R,2}, Q^{R,+}\right\}_\mathrm{P} = Q^{R,+}\cosh\left(\frac{\sqrt{C}}{2}Q^{R,-}\right)$$

Right Lax pair, classical *r*-matrix

$$L^R_t(\lambda_R) = \frac{1}{1-\lambda_R^2}\Big[-T^+\left(-J^-_t + \lambda_R J^-_x\right) + T^2\left(-J^2_t + \lambda_R J^2_x + \sqrt{C}\lambda_R J^-_t - \sqrt{C}\lambda_R^2 J^-_x\right)$$
$$-T^-\left(-J^+_t + \lambda_R J^+_x + \sqrt{C}\lambda_R J^2_t - \sqrt{C}\lambda_R^2 J^2_x + C\lambda_R J^-_x - C\lambda_R^2 J^-_t\right)\Big]$$

$$L^R_x(\lambda_R) = \frac{1}{1-\lambda_R^2}\Big[-T^+\left(-J^-_x + \lambda_R J^-_t\right) + T^2\left(-J^2_x + \lambda_R J^2_t + \sqrt{C}\lambda_R J^-_x - \sqrt{C}\lambda_R^2 J^-_t\right)$$
$$-T^-\left(-J^+_x + \lambda_R J^+_t + \sqrt{C}\lambda_R J^2_x - \sqrt{C}\lambda_R^2 J^2_t + C\lambda_R J^-_t - C\lambda_R^2 J^-_x\right)\Big]$$

λ_R : spectral parameter

→ null deformed rational *r*-matrix (≠trigonometric)

Note With a certain gauge-transformation, $L^R_\mu(\lambda_R)$ can be written with a flat and conserved current j_μ:

$$L^R_t(\lambda_R) = G^{-1}\left(\frac{1}{1-\lambda_R^2}\left[j_t - \lambda_R j_x\right]\right)G - G^{-1}\partial_t G$$

$$L^R_x(\lambda_R) = G^{-1}\left(\frac{1}{1-\lambda_R^2}\left[j_x - \lambda_R j_t\right]\right)G - G^{-1}\partial_x G$$

$$G = \mathrm{e}^{-\sqrt{C}T^2\chi}\mathrm{e}^{-\sqrt{C}T^-\xi} \qquad \xi(x) = \frac{1}{2}\int_{-\infty}^{\infty} dy\ \epsilon(x-y)j^{R,2}_t(y)$$

A (non-local) flat and conserved current in right description:

$$j_\mu = -T^+\epsilon_{\mu\nu}\partial^\nu\left(\frac{1}{\sqrt{C}}\mathrm{e}^{-\sqrt{C}\chi}\right) + T^2\epsilon_{\mu\nu}\partial^\nu\left(\xi\mathrm{e}^{-\sqrt{C}\chi}\right)$$
$$-T^-\left[j^{R,+}_\mu + \frac{\sqrt{C}}{2}\epsilon_{\mu\nu}\partial^\nu\left(\xi^2\mathrm{e}^{-\sqrt{C}\chi}\right)\right]$$

Enhancement of *q*-deformed Poincare symmetry

BIZZ construction → Infinite number of conserved charges $Q_{(0)}\,,\ Q_{(1)}\,, \cdots$

Note An expansion of $U^R(\lambda_R)$ also gives $Q_{(n)}$.

Note $Q_{(0)}$ contains the generators of *q*-deformed Poincare symm.

$$Q^-_{(0)} = \frac{2}{\sqrt{C}}\sinh\left(\frac{\sqrt{C}}{2}Q^{R,-}\right)\,,\quad Q^2_{(0)} = \cosh\left(\frac{\sqrt{C}}{2}Q^{R,-}\right)Q^{R,2}$$

$$Q^+_{(0)} = Q^{R,+} + \frac{\sqrt{C}}{4}\sinh\left(\frac{\sqrt{C}}{2}Q^{R,-}\right)\left(Q^{R,2}\right)^2$$

The algebra of $Q_{(n)}$ should be a 1-para. deformation of SL(2,R) Yangian.

C→0 limit ↔ Yangian limit

Future works

- Algebra of enhanced *q*-deformed Poincare symmetry
- Left-Right duality [I.K., T.Matsumoto, K.Yoshida, 1203.3400]