MELLIN BOOTSTRAP – A FAQ SESSION (ASCONA, JUL '17)

I will describe an alternative approach to the conf. bootstrap that dates back to Polyakov ('74) but in a modern reincarnation. It ~~might~~ appears to give, in this new guise some significant calculational advantages (mention later). But since there aren't too many bootstrap people today here, broad sweep behind. I would like, however, to concentrate on the ~~conceptual basis of~~ this approach and its "meaning" (in a larger context of AdS/CFT) – to get ~~ideas back~~ feedback. To ~~provoke~~ stimulate discussion/questions I have organised this as a FAQ session. (work w/ A. KAVIRAJ, K. SEN + A. SINHA).

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*. "What's different?"

– Built in crossing symm. → Require compatibility w/ OPE
– Use a basis of Witten diagrams $W_{\Delta,\ell}(u,v)$ rather than conf. bl. $G_{\Delta,\ell}(u,v)$
– Work in Mellin space $W_{\Delta,\ell}(u,v) \to M_{\Delta,\ell}(s,t)$. → Much more explicit.

Now explain each in turn.

* "~~What are the~~ How does the OPE compatibility impose constraints ~~from the OPE~~?"

$$A(u,v) = \sum_{\Delta,\ell} c_{\Delta,\ell}\left(W^{(s)}_{\Delta,\ell}(u,v) + W^{(t)}_{\Delta,\ell}(u,v) + W^{(u)}_{\Delta,\ell}(u,v)\right)$$

⇒ Spurious powers of (u) for e.g. Set them to zero.

*. "Why Witten diag?"

– We know that Feynman diagrams of a (local) field theory is the simplest way to obtain solutions of dispersion relations for amplitudes. The analogous conditions in CFT descr. of unitarity and conf. inv. "building blocks" led Polyakov to construct them from pieces which we now better understand to be Witten diagrams – following from a field theory in AdS.

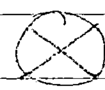
– (recall) $W^{(s)}_{\Delta,\ell}(u,v) = N_{\Delta,\ell}\, G^{(s)}_{\Delta,\ell}(u,v) + \sum_n N_{n,\ell}\, G^{(s)}_{\Delta_n,\ell}(u,v)$ — additional; need to / will cancel out in the sum.

$\Delta_n = 2\Delta_\phi + 2n + \ell$ = dim of "$\mathcal{O}\,\partial^{2n}\partial^{\ell}\,\mathcal{O}$" – double trace ops.

– Witten diag. are better behaved in Mellin space than conf. bl.. Presumably, a better (more convergent) basis.

* Are we assuming an AdS dual?
- No. A kinematical basis to expand in. Applies to any CFT
- without a large N or semi-classical spacetime limit.

* Aren't double trace ops. physical? Why cancel them?
- Physical double trace ops. are added in the sum over all primaries (Δ, ℓ). In general they will not have dim. $2\Delta_\phi + 2n + \ell$. We are not making an expansion in which these are the leading terms. We are thus cancelling the contrib. of "fictional" double-trace ops. of dim. $2\Delta_\phi + 2n + \ell$.

* Contact Witten diagrams? 
- It appears we need to add the particular ones needed (almost conserved) for a local field theory. E.g. for higher spin currents ⇔ massless gauge fields, the terms dictated by closure of higher spin gauge symmetry. Can be absorbed in a redefinition of exchange vertices.

* Why Mellin space?
(a) $W^{(s)}_{\Delta,\ell}(u,v) \to M^{(s)}_{\Delta,\ell}(s,t)$ much simpler.
(b) Mellin amplitudes are meromorphic (like scattering ampl.)
(c) Can make a partial wave expansion.

* What do Witten diag. look like in Mellin space.
$B^{(s)}_{\Delta,\ell}(s,t)$ and then $M^{(s)}_{\Delta,\ell}(s,t)$.

*. What are the cancellation conditions?

* How does the partial wave expansion help?

*. Show me the money! (Results)

$$\Delta_\phi = 1 - \frac{\epsilon}{2} + \frac{1}{108}\epsilon^2 + \frac{109}{11664}\epsilon^3 \qquad \frac{C_{\phi^2}}{C_{free}} = 1 - \frac{1}{3}\epsilon - \frac{17}{81}\epsilon^2.$$

$$\Delta_{\phi^2} = 2 - \frac{2}{3}\epsilon + \frac{19}{162}\epsilon^2 + (\quad)\epsilon^3.$$
$\hookrightarrow \zeta(3)$

$$\Delta_\ell = d-2+\ell + \left(1 - \frac{6}{\ell(\ell+1)}\right)\frac{\varepsilon^2}{54} + \delta_\ell^{(3)}\varepsilon^3$$

$$\frac{C_\ell}{C_{free}} = 1 + \frac{\varepsilon^2}{54\ell(\ell+1)}\left[6(\ell+1)^{-1} + 2(\ell^2+\ell-3)H_\ell - H_{2\ell}(\ell-2)(\ell-3)\right] + c_\ell^{(3)}\varepsilon^3$$

$$C_T = \frac{d^2\Delta_\phi^2}{(d-1)^2 C_2} \implies \frac{C_T}{C_{free}} = 1 - \frac{5\varepsilon^2}{324} - \frac{233\varepsilon^3}{8748}$$

for $\varepsilon = 1$, 3d Ising model $\left.\frac{C_T}{C_{free}}\right|_{num} = 0.946534\,(11)$

$$\left.\frac{C_T}{C_{free}}\right|_{\varepsilon=1} \simeq 0.957933$$

Also leading order contributions to ϕ^4, spin 2 twist 4, etc. from looking at higher order poles.

MELLIN BOOTSTRAP - A FAQ Session

(w/ A. Kaviraj, K. Sen, A. Sinha; w/ A. Sinha)

4 identical scalar ops — Δ_ϕ.

$$A(u,v) \rightarrow \mathcal{M}(s,t)$$

$$= \int_{-i\infty}^{i\infty} ds\, dt\, u^s v^t \rho_{\Delta_\phi}(s,t) \mathcal{M}(s,t)$$

$$\rho_{\Delta_\phi}(s,t) = \Gamma^2(\Delta_\phi - s)\Gamma^2(-t)\Gamma^2(s+t)$$

* WHAT'S DIFFERENT?

ⓐ Built in crossing symmetry

$\xrightarrow{?}$ compatible w/ the OPE?

ⓑ Use a basis of Witten diagrams

$W^{(s)}_{\Delta,\ell}(u,v)$ instead of conf. bl.

$G^{(s)}_{\Delta,\ell}(u,v)$

ⓒ Work in Mellin space

* How do we impose the constraints from OPE?

$$A(u,v) = \sum_{\Delta \ell} C_{\Delta,\ell} \left(W^{(s)}_{\Delta\ell}(u,v) + (t,u) \right)$$

$\sum_{\Delta\ell}$: over all physical primaries

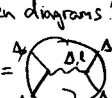
$C_{\Delta,\ell}$: prop to OPE coeffs

Require that an expansion in u have only physical op dimensions

$u^{\Delta_\phi n}$,

$u^{\Delta_\phi n} \ln u$

* Why Witten diagrams?

$W^{(s)}_{\Delta,\ell}(u,v) =$ 

"Fictional" double trace op.s w/ $2\Delta_\phi + 2n + \ell$.

" $O_\phi \partial^{2n} \partial^\ell O_\phi$ "

(a) Obeys unitarity + Conf invariance.

(b) $W^{(s)}_{\Delta,\ell}(u,v) = N_{\Delta,\ell}\, G^{(s)}_{\Delta,\ell}(u,v) + \sum_{n=1}^{\infty} N_{\Delta_n,\ell}\, G^{(s)}_{\underset{\sim}{\Delta_n},\ell}(u,v)$

(c) They are very nicely behaved in Mellin space.

* Are we assuming an AdS dual?

No.

* $\bigotimes$?

* Why Mellin space?

(a) $W^{(s)}_{\Delta,\ell}(u,v) \longrightarrow M^{(s)}_{\Delta,\ell}(s,t)$ → much simpler

(b) Behave like scatt. amp

(c) Can make a partial wave exp.

* Cancellation of spurious powers of u → cancellation of spurious poles → $\frac{\alpha^{(1)}(t)}{s-\Delta_\phi-n}, \frac{\alpha^{(2)}(t)}{(s-\Delta_\phi-n)^2}$

$$\Delta_\phi = 1 + (\ )\varepsilon + (\ )\varepsilon^2 + (\ )\varepsilon^3$$

$$\Delta_{\phi^2} = 2 + (\ )\varepsilon + (\ )\varepsilon^2$$

$$\Delta_\ell = (d-2+\ell) + \ldots + (\ )\varepsilon^3$$

$$C_{\phi\phi\phi^2} = C_0 + C_{\text{free}} + \ldots \varepsilon^3$$

$$C_{\phi\phi J^\ell} = C_\ell + C^{(\ell)}_{\text{free}} \quad (\ )\varepsilon^3$$

$$\frac{C_T}{C^T_{\text{free}}} = 1 - \frac{5\varepsilon^2}{324} - \frac{233\varepsilon^3}{8748},$$

New!

$$G^{(s)}(u,v) \longrightarrow B^{(s)}_{\Delta,\ell}(s,t)\,;\quad W^{(s)}_{\Delta,\ell}(u,v) \to M^{(s)}_{\Delta,\ell}(s,t)$$

$$B^{(s)}_{\Delta,\ell}(s,t) = \left(e^{i\pi(2s+\Delta+\ell-2h)} - 1\right) \frac{\Gamma\left(\frac{\Delta-\ell}{2}-s\right)\Gamma\left(\frac{2h-\Delta-\ell}{2}-s\right)}{\Gamma^2(\Delta_\phi - s)} \times P^{(s)}_{\Delta-h,\ell}(s,t) = \sum_{m=0}^{\infty} \frac{\tilde{c}_m\, Q^{\Delta}_{\ell,m}(t)}{s - \frac{(\Delta-\ell)}{2} - m} + \ldots$$

$(h = d/2)$

$$M^{(s)}_{\Delta\ell}(s,t) = {}_3F_2\left[\frac{\Delta-\ell}{2}-s,\ldots,1+\frac{\Delta-\ell}{2}-s,\ldots,1\right] \times P^{(s)}_{\Delta-h,\ell}(s,t) \times \left(\mathrm{Poly}_{\ell-1}(s,t)\right) = \sum_{m=0}^{\infty} \frac{\tilde{c}_m\, Q^{\Delta}_{\ell m}(t)}{s - \frac{(\Delta-\ell)}{2} - m} + \mathrm{Pol}_{\ell-1}$$