From Higher Spins to Strings

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Based on: M. R. Gaberdiel and R. G.
(arXiv:1406.6103 and also 1305.4181)

The Bird's Eye-View

- *Some very brief motivation.*
- *Punchline: How spectrum and symmetries of symmetric product CFT organises itself in terms of HS symmetry.*
- *The large $\mathcal{N}=4$ coset: a nodding acquaintance - its spectrum and symmetries and the dual Vasiliev theory.*
- *Contraction to a small $\mathcal{N}=4$ CFT: the continuous orbifold $(T^4)^{N+1}/U(N)$.*
- *The relation to the symmetric product $(T^4)^{N+1}/S_{N+1}$ and how it repackages the stringy chiral algebra.*

Why are We Studying Higher Spin Theories?

- *Free YM theory has a tower of conserved currents dual to Vasiliev H-spin gauge fields (*Sundborg, Witten*).*
- *Signals the presence of a large unbroken symmetry phase of the string theory (*Gross, Witten, Moore, Sagnotti et.al.*).*
- *Can the Vasiliev H-Spin symmetries help to get a handle on the extended stringy symmetry in the tensionless limit?*
- AdS_3 *might be a good test case since it already has Virasoro (and then extended to* W_∞ *-* Henneaux-Rey, Campoleoni et.al.*).*
- *Symmetric product CFT for D1-D5 system has been believed to be dual to tensionless limit of string theory.*

The Punchline

Vasiliev higher spin symmetry organises all the states of the $(T^4)^{N+1}/S_{N+1}$ *orbifold symmetric product CFT = Tensionless limit of strings on* $AdS_3 \times S^3 \times T^4$

Stringy Symmetries

In particular:

The chiral sector (conserved currents) can be written in terms of specific representations of the higher spin symmetry algebra.

$$\mathcal{Z}_{NS}(q,y) = \sum_{\Lambda \in U(N)} n(\Lambda) \chi_{(0;\Lambda)}(q,y)$$

Chiral part of Symm. Prod.

multiplicity of S_{N+1} singlets in Λ

Characters of $\mathcal{N}=4$ minimal model coset: W_∞ reps.

Infinite (stringy) extension of W_∞ symmetry.

Explicitly.....

- *The vacuum character ($\Lambda = 0$) contains the usual W_∞ generators - bilinears in free fermions and bosons.*
- *Additional chiral generators ($\Lambda \neq 0$) can be written down explicitly in terms of free fermions and bosons.*

$$\Lambda = [2, 0 \ldots, 0] \leftrightarrow \sum_{i=1}^{N+1} \psi^{i\alpha}_{-1/2} \psi^{i\beta}_{-1/2}$$

$$\Lambda = [0, 2, 0 \ldots, 0] \leftrightarrow \sum_{i,j=1}^{N+1} \psi^{i\alpha}_{-1/2} \psi^{j\beta}_{-1/2} \psi^{i\gamma}_{-1/2} \psi^{j\delta}_{-1/2}$$

Large $\mathcal{N} = 4$

- *String theory on* $AdS_3 \times S^3 \times T^4$ *has small* $\mathcal{N} = 4$ *SUSY .*
- *Useful to consider via a limit of H-spin holography for large* $\mathcal{N} = 4$ *coset CFTs.* (Gaberdiel-R.G.)
- *Large* $\mathcal{N} = 4$ *SCA has two* SU(2) *Kac-Moody algebras.* *Thus labelled by one extra parameter:* $\gamma = \dfrac{k_-}{k_+ + k_-}$.
- *Small* $\mathcal{N} = 4$ *obtained as a contraction -* $k_+ \to \infty$.
- *Only one* SU(2) *KM algebra at level* k_- .

Large $\mathcal{N} = 4$ Coset Holography

The CFT:

4(N+1) *free fermions*

$$\frac{\mathfrak{su}(N+2)^{(1)}_{\kappa}}{\mathfrak{su}(N)^{(1)}_{\kappa} \oplus \mathfrak{u}(1)^{(1)}} \oplus \mathfrak{u}(1)^{(1)} \cong \frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N+4)_1}{\mathfrak{u}(N)_{k+2}} \oplus \mathfrak{u}(1) \ .$$

$c = \frac{6(k+1)(N+1)}{k+N+2}$. *Take 't Hooft limit* $N, k \to \infty$ *with* $\lambda = \frac{N+1}{N+k+2} = \gamma$ *fixed*. (Gaberdiel-R.G.)

Has Large $\mathcal{N} = 4$ *(*van Proeyen et.al., Sevrin et.al.*) with*

$k_+ = (k+1); k_- = (N+1)$

Coset Holography (Contd.)

The H-Spin Dual:

- *Vasiliev theory based on* $shs_2[\lambda]$ *gauge group* (Prokushkin-Vasiliev).
- *One higher spin gauge supermultiplet for each spin* $s \geq 1$

SU(2) labels

$$
R^{(s)}: \quad \begin{array}{ll}
s: & (\mathbf{1},\mathbf{1}) \\
s+\frac{1}{2}: & (\mathbf{2},\mathbf{2}) \\
s+1: & (\mathbf{3},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3}) \\
s+\frac{3}{2}: & (\mathbf{2},\mathbf{2}) \\
s+2: & (\mathbf{1},\mathbf{1}) \ .
\end{array}
$$

- *Generates an asymptotic super* W_∞ *algebra which matches nontrivially with coset (*Gaberdiel-Peng, Beccaria et.al.*).*

W_∞ Representations

- *Primaries labelled by* $(\Lambda_+; \Lambda_-, u)$

$\in \mathfrak{su}(N+2)_k$ $\quad \in \mathfrak{su}(N)_{k+2}$ $\quad \in \mathfrak{u}(1)_\kappa$ *(will be omitted)*

- (0;f) $\leftrightarrow$ *"Perturbative" matter multiplets of H-Spin theory (with* $(0;\Lambda) \leftrightarrow$ *multi-particles) (*Chang-Yin*).*

$$h(0;\mathrm{f}) = \frac{k+\frac{3}{2}}{N+k+2} \rightarrow \frac{1-\lambda}{2}$$

$$\mathcal{H}^{(\mathrm{pert})} = \bigoplus_{\Lambda} (0;\Lambda) \otimes \overline{(0;\Lambda^*)} \subset \mathcal{H}^{(\mathrm{diag})} = \bigoplus_{\Lambda_+,\Lambda_-} (\Lambda_+;\Lambda_-) \otimes \overline{(\Lambda_+^*;\Lambda_-^*)}$$

Contains "light states"

$\mathcal{N} = 4 \xrightarrow{\text{contracts}} \mathcal{N} = 4$

$$c = \frac{6(k+1)(N+1)}{k+N+2} \xrightarrow{k\to\infty} c = 6(N+1)$$

- *Coset CFT reduces to a continuous orbifold* $(T^4)^{N+1}/U(N)$.
- *The WZW factors decompactify to give* 4(N+1) *free bosons which combine with the* 4(N+1) *free fermions, gauged by U(N)*.

Bosons: $2\cdot(\mathbf{N},\mathbf{1})\oplus 2\cdot(\bar{\mathbf{N}},\mathbf{1})\oplus 4\cdot(\mathbf{1},\mathbf{1})$

Fermions: $(\mathbf{N},\mathbf{2})\oplus(\bar{\mathbf{N}},\mathbf{2})\oplus 2\cdot(\mathbf{1},\mathbf{2})$

fund. of U(N)

$SU(2)_R$

Singlet of U(N)

Continuous Orbifold

- *Untwisted sector: U(N) singlets formed from fermions/bosons.*

  $E.g.\ (0;\bar{\mathrm{f}}) \otimes \overline{(0;\mathrm{f})} \leftrightarrow \psi^{\bar{i}\alpha}\widetilde{\psi}^{i\beta}$; (*Note:* $h(0;f) = \frac{1-\lambda}{2} \xrightarrow{k\to\infty} \frac{1}{2}$)

- *More generally,*

Vasiliev States

$$\mathcal{H}_{\text{untwisted}} = \bigoplus_{\Lambda} (0;\Lambda) \otimes \overline{(0;\Lambda^*)} = \mathcal{H}^{(\text{pert})}$$

Similar to bosonic and $\mathcal{N}=2$ cases

(Gaberdiel-Suchanek, Gaberdiel-Kelm)

- *Twisted Sector: Continuous twists* (U(N) holonomies) *leads to a continuum (incl. light states). Labelled by* $(\Lambda_+;\Lambda_-)$: $w/\ \Lambda_+ \neq 0$.

A Tale of Two Orbifolds

- *How do we relate* $(T^4)^{N+1}/U(N)$ *to* $(T^4)^{N+1}/S_{N+1}$ *?*
- $S_{N+1} \subset U(N)$ *and* $\mathbf{N}, \bar{\mathbf{N}} \to N$ ← *N Dim. Irrep. of* S_{N+1}

Bosons: $2 \cdot (\mathbf{N}, \mathbf{1}) \oplus 2 \cdot (\bar{\mathbf{N}}, \mathbf{1}) \oplus 4 \cdot (\mathbf{1}, \mathbf{1}) \to 4 \cdot (N, \mathbf{1}) \oplus 4 \cdot (1, \mathbf{1})$

Fermions: $(\mathbf{N}, \mathbf{2}) \oplus (\bar{\mathbf{N}}, \mathbf{2}) \oplus 2 \cdot (\mathbf{1}, \mathbf{2}) \to 2 \cdot (N, \mathbf{2}) \oplus 2 \cdot (1, \mathbf{2})$

How fermions and bosons in usual symmetric product orbifold transform

$$\Rightarrow (T^4)^{N+1}/U(N)\Big|_{\text{untwisted}} \subset (T^4)^{N+1}/S_{N+1}\Big|_{\text{untwisted}}$$

Two Orbifolds (Contd.)

- *Therefore:*

$$\mathcal{H}^{(\mathrm{pert})} = \bigoplus_{\Lambda} (0;\Lambda) \otimes \overline{(0;\Lambda^*)} \subset \mathcal{H}^{(\mathrm{Sym.Prod.})}\Big|_{\mathrm{untwisted}}$$

- *i.e. Vasiliev states are a closed subsector of the Symmetric Product CFT = Tensionless string theory.*

- *More generally, states of the symmetric product CFT must transform in specific representations of the chiral algebra of the continuous orbifold (the U(N) invariant i.e.* W_∞ *currents).*

$$Z_{\mathrm{NS}}(q,\bar{q},y,\bar{y}) = |\mathcal{Z}_{\mathrm{vac}}(q,y)|^2 + \sum_j |\mathcal{Z}_j^{(\mathrm{U})}(q,y)|^2 + \sum_\beta |\mathcal{Z}_\beta^{(\mathrm{T})}(q,y)|^2$$

Other untwisted sectors

Twisted sectors

Stringy Chiral Algebra

- *The vacuum sector (S_{N+1} invariant currents) can therefore be organised in terms of coset (W_∞) representations - from the untwisted sector of the continuous orbifold.*

$$\mathcal{Z}_{\rm vac}(q,y) = \sum_{\Lambda\in U(N)} n(\Lambda)\chi_{(0;\Lambda)}(q,y)$$

- *Each such representation comes with a multiplicity which would be given by the number of times the singlet of S_{N+1} appears in the U(N) representation Λ.*

- *A vast extension of W_∞ - generators not just bilinear in fermions/bosons but also cubic, quartic etc.*

Reality Check

- *Explicitly verify this equality to low orders - use DMVV prescription to compute*

$$
\begin{aligned}
\mathcal{Z}_{\rm vac}(q,y) \quad = \quad & 1 + \left(2y + 2y^{-1}\right)q^{\frac{1}{2}} + \left(2y^2 + 12 + 2y^{-2}\right)q \\
& + \left(2y^3 + 32y + 32y^{-1} + 2y^{-3}\right)q^{\frac{3}{2}} \\
& + \left(2y^4 + 52y^2 + 159 + 52y^{-2} + 2y^{-4}\right) q^2 \\
& + \left(2y^5 + 62y^3 + 426y + 426y^{-1} + 62y^{-3} + 2y^{-5}\right) q^{\frac{5}{2}} \\
& + \left(2y^6 + 64y^4 + 767y^2 + 1800 + 767y^{-2} + 64y^{-4} + 2y^{-6}\right) q^3 \\
& + O(q^{\frac{7}{2}}) \ .
\end{aligned}
$$

It Agrees!

Vasiliev higher spin fields

Additional higher spin generators : $\sum_i \psi^{i\alpha}_{-\frac{1}{2}} \psi^{i\beta}_{-\frac{1}{2}}$

$$
\begin{aligned}
\mathcal{Z}_{\rm vac}(q,y) \quad = \quad & \chi_{(0;0)}(q,y) + \chi_{(0;[2,0,\ldots,0])}(q,y) + \chi_{(0;[0,0,\ldots,0,2])}(q,y) \\
& + \chi_{(0;[3,0,\ldots,0,0])}(q,y) + \chi_{(0;[0,0,0,\ldots,0,3])}(q,y) + \chi_{(0;[2,0,\ldots,0,1])}(q,y) \\
& + \chi_{(0;[1,0,0,\ldots,0,2])}(q,y) + 2\cdot\chi_{(0;[4,0,\ldots,0,0])}(q,y) + 2\cdot\chi_{(0;[0,0,0,\ldots,0,4])}(q,y) \\
& + \chi_{(0;[0,2,0,\ldots0,0])}(q,y) + \chi_{(0;[0,0,\ldots0,2,0])}(q,y) + \chi_{(0;[3,0,\ldots,0,1])}(q,y) \\
& + \chi_{(0;[1,0,0,\ldots,0,3])}(q,y) + 2\cdot\chi_{(0;[2,0,0,\ldots,0,2])}(q,y) + \chi_{(0;[1,2,0,\ldots,0])}(q,y) \\
& + \chi_{(0;[0,\ldots,0,2,1])}(q,y) + \chi_{(0;[2,1,0,\ldots,0,1])}(q,y) + \chi_{(0;[1,0,\ldots,0,1,2])}(q,y) \\
& + \chi_{(0;[0,2,0,\ldots,0,1])}(q,y) + \chi_{(0;[1,0,\ldots,0,2,0])}(q,y) + 3\cdot\chi_{(0;[3,0,\ldots,0,2])}(q,y) \\
& + 3\cdot\chi_{(0;[2,0,\ldots,0,3])}(q,y) + \chi_{(0;[1,1,0,\ldots,0,2])}(q,y) + \chi_{(0;[2,0,\ldots,0,1,1])}(q,y) \\
& + \chi_{(0;[0,0,2,0,\ldots,0])}(q,y) + \chi_{(0;[0,\ldots,0,2,0,0])}(q,y) + 3\cdot\chi_{(0;[0,2,0,\ldots,0,2])}(q,y) \\
& + 3\cdot\chi_{(0;[2,0,\ldots,0,2,0])}(q,y) + \chi_{(0;[1,1,0,\ldots,0,1,1])}(q,y) + \mathcal{O}(q^{7/2})\ .
\end{aligned}
$$

Reality Check (Contd.)

- *Can do something similar for the simplest non-trivial untwisted sector - which contains 16 of the 20 marginal ops.*

$$\mathcal{Z}_1^{(\mathrm{U})}(q,y) = \sum_{\Lambda} n_1(\Lambda)\, \chi_{(0;\Lambda)}(q,y)$$

Contains $\psi^{i\alpha}_{-\frac{1}{2}}$

Multiplicity of N dim. irrep of S_{N+1} in Λ

- *Compute LHS*

$$\begin{aligned}
\mathcal{Z}_1(q,y) \quad = \quad & (2y+2y^{-1})q^{1/2} + (5y^2+16+5y^{-2})q^1 \\
& + (6y^3+58y+58y^{-1}+6y^{-3})q^{3/2} \\
& + (6y^4+128y^2+315+128y^{-2}+6y^{-4})q^2 \\
& + (6y^5+198y^3+1030y+1030y^{-1}+198y^{-3}+6y^{-5})q^{5/2} \\
& + (6y^6+240y^4+2290y^2+4724+2290y^{-2}+240y^{-4}+6y^{-6})q^3 \\
& + \mathcal{O}(q^3)\ .
\end{aligned}$$

Agrees too....

(0;f) contribution

$$
\begin{aligned}
\mathcal{Z}_1(q,y) \;=\;& \chi_{(0;[1,0,\ldots,0])}(q,y) + \chi_{(0;[0,\ldots,0,1])}(q,y) + \chi_{(0;[1,0,\ldots,0,1])}(q,y) \\
&+ \chi_{(0;[2,0,\ldots,0])}(q,y) + \chi_{(0;[0,0,\ldots,0,2])}(q,y) + \chi_{(0;[1,1,0\ldots,0])}(q,y) \\
&+\chi_{(0;[0,\ldots,0,1,1])}(q,y) + 2\cdot\chi_{(0;[2,0,\ldots,0,1])}(q,y) + 2\cdot\chi_{(0;[1,0,0,\ldots,0,2])}(q,y) \\
&+ \chi_{(0;[0,2,0,\ldots0,0])}(q,y) + \chi_{(0;[0,0,\ldots0,2,0])}(q,y) + 2\cdot\chi_{(0;[3,0,\ldots,0,0])}(q,y) \\
&+2\cdot\chi_{(0;[0,0,0,\ldots,0,3])}(q,y) + 2\cdot\chi_{(0;[1,1,0\ldots,0,1])}(q,y) + 2\cdot\chi_{(0;[1,0,\ldots,0,1,1])}(q,y) \\
&+ 5\cdot\chi_{(0;[2,0,\ldots,0,2])}(q,y) + \chi_{(0;[0,1,0\ldots,0,2])}(q,y) + \chi_{(0;[2,0,\ldots,0,1,0])}(q,y) \\
&+ 2\cdot\chi_{(0;[2,1,0,\ldots,0])}(q,y) + 2\cdot\chi_{(0;[0,\ldots,0,1,2])}(q,y) + \chi_{(0;[0,1,1,0,\ldots,0])}(q,y) \\
&+\chi_{(0;[0,\ldots,0,1,1,0])}(q,y) + 3\cdot\chi_{(0;[0,2,0,\ldots,0,1])}(q,y) + 3\cdot\chi_{(0;[1,0,\ldots,0,2,0])}(q,y) \\
&+ 4\cdot\chi_{(0;[3,0,\ldots,0,1])}(q,y) + 4\cdot\chi_{(0;[1,0,0,\ldots,0,3])}(q,y) + 5\cdot\chi_{(0;[1,1,0,\ldots,0,2])}(q,y) \\
&+5\cdot\chi_{(0;[2,0,\ldots,0,1,1])}(q,y) + \chi_{(0;[0,1,0\ldots,0,1,1])}(q,y) + \chi_{(0;[1,1,0,\ldots,0,1,0])}(q,y) \\
&+ 3\cdot\chi_{(0;[4,0,\ldots,0,0])}(q,y) + 3\cdot\chi_{(0;[0,0,0,\ldots,0,4])}(q,y) + 3\cdot\chi_{(0;[1,2,0,\ldots,0])}(q,y) \\
&+3\cdot\chi_{(0;[0,\ldots,0,2,1])}(q,y) + \chi_{(0;[0,0,2,0,\ldots,0])}(q,y) + \chi_{(0;[0,\ldots,0,2,0,0])}(q,y) \\
&+ 4\cdot\chi_{(0;[2,1,0,\ldots,0,1])}(q,y) + 4\cdot\chi_{(0;[1,0,\ldots,0,1,2])}(q,y) + 2\cdot\chi_{(0;[0,1,1,0,\ldots,0,1])}(q,y) \\
&+2\cdot\chi_{(0;[1,\ldots,0,1,1,0])}(q,y) + \chi_{(0;[1,0,1,0,\ldots,0,2])}(q,y) + \chi_{(0;[2,0,\ldots,0,1,0,1])}(q,y) \\
&+ 7\cdot\chi_{(0;[0,2,0,\ldots,0,2])}(q,y) + 7\cdot\chi_{(0;[2,0,\ldots,0,2,0])}(q,y) + 9\cdot\chi_{(0;[3,0,\ldots,0,2])}(q,y) \\
&+9\cdot\chi_{(0;[2,0,\ldots,0,3])}(q,y) + 2\cdot\chi_{(0;[0,1,0,\ldots,0,2,0])}(q,y) + 2\cdot\chi_{(0;[0,2,0,\ldots,0,1,0])}(q,y) \\
&+ 2\cdot\chi_{(0;[0,1,0\ldots,0,3])}(q,y) + 2\cdot\chi_{(0;[3,0,\ldots,0,1,0])}(q,y) + 6\cdot\chi_{(0;[1,1,0,\ldots,0,1,1])}(q,y) \\
&+ \mathcal{O}(q^{7/2})\,,
\end{aligned}
$$

Twisted Sector

- *A similar reorganisation also works for the* twisted sectors *of the symmetric product.*
- *Have studied the* 2-cycle *twisted sector - contains the other four marginal operators.*

$$\mathcal{Z}^{(2)}_{\pm}(q,y) = \sum_{\Lambda'_-, l_0; \mp 1} \widetilde{n}(\Lambda'_-)\, \chi_{([\frac{k}{2},0\ldots,0];[\frac{k}{2}+l_0,\Lambda'_-])}(q,y)$$

Multiplicity of S_{N-1} *singlets in* Λ'_-

- *Again explicit answers check.*

Miscellaneous Remarks

- *Can also refine this organisation of the spectrum into single and multiparticle states*.

- *If we associate the free fermions/bosons with Cartan elements of an adjoint valued field (* $\phi_i \to \Phi_{ii}$ *), then additional single particle currents which are higher order polynomials*.

$$\sum_{i=1}^{N+1} \phi_i^4 \sim \mathrm{Tr}\Phi^4$$

- *Higher Regge trajectories compared to the leading one - Vasiliev states*.

- *Note, no light states (as for adjoint theories)* $\longleftrightarrow$ *not local w.r.t. stringy chiral algebra (non-diagonal modular invariant)*.

A Slogan

$$Vector/HS\ holography = U(N)\ orbifold$$

whereas

$$Matrix/Stringy\ holography = symm.\ orbifold$$

Looking Back

Essentially we have:

- *Identified the Vasiliev states as a subsector of the symmetric product CFT.*
- *Assembled the full spectrum of the tensionless string theory in terms of representations of the super W_∞ algebra.*
- *Characterised the full set of massless higher spin states at this point in terms of W_∞ representations - a huge unbroken stringy symmetry algebra.*

Looking Ahead

- *Understand the higgsing of the stringy symmetries in going away from the tensionless point (deforming by marginal op.).*
- *Does it constrain the spectrum, 3-point functions ? (Use philosophy of Wigner-Eckart)*
- *Is there a relation to integrability in the underlying worldsheet theory?*
- *Can one understand the string theory on* $AdS_3 \times S^3 \times S^3 \times S^1$ *in a similar way?*
- *Relation to ABJ triality (*Chang et.al.*) and to proposal for free super Yang-Mills spectrum (*Beisert, Bianchi et.al. *), and multiparticle HS algebra (*Vasiliev*) ?*

Thank You