Nonabelian $(2,0)$ Tensor Multiplets and 3-algebras

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(with Neil Lambert, arXiv:1007.2982)

Motivation

Over the last two years there has been significant amount of work on actions for multiple M2-branes.

Progress relied on the introduction of a novel algebraic structure: a 3-algebra. [Bagger-Lambert, Gustavsson]

This is defined through

$$[T^A, T^B, T^C] = f^{ABC}{}_D T^D$$

and satisfies the 'fundamental identity'

$$f^{[ABC}{}_E f^{D]EF}{}_G = 0\ .$$

These ideas solidified in the ABJM proposal for bifundamental Chern-Simons-matter theory describing N M2-branes on a $\mathbb{C}^4/\mathbb{Z}_k$ M-theory singularity.
[Aharony-Bergman-Jafferis-Maldacena]

Many developments in $\mathrm{AdS}_4/\mathrm{CFT}_3$...

3-algebra description not necessary but possible.
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But what about the M5-brane??

It’s complicated: cannot even get Lagrangian for single M5 because of selfdual three-form field strength.

Note: Many indirect ways of attacking this like sacrificing manifest 6d Lorentz invariance, introducing new scalar field, no selfduality at Lagrangian but directly at path integral. [Aganagic-Park-Popescu-Schwarz, Pasti-Sorokin-Tonin, Bandos et al., Cederwall-Nilsson-Sundell]

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⇒ Assume that 3-algebra approach has deeper connections to M-theory and use a similar approach to M2-brane.

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- This 6d theory involves 3-algebras
- No direct evidence of a multiple M5-brane interpretation
- Has some interesting features

Outline

- Set-up of the calculation
- Susy closure
- Spacelike reduction
- Null reduction
- Summary

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- Start with the susy transformations for the abelian M5-brane
- Generalise this to allow for nonabelian fields and interactions
- Investigate the closure of the susy algebra
- Obtain e.o.m. and constraints
- Interpret the result

The susy transformations for the free 6d $(2,0)$ tensor multiplet are

$$\begin{aligned}
\delta X^I &= i\bar{\epsilon}\Gamma^I\Psi \\
\delta\Psi &= \Gamma^\mu\Gamma^I\partial_\mu X^I\epsilon + \frac{1}{3!}\frac{1}{2}\Gamma^{\mu\nu\lambda}H_{\mu\nu\lambda}\epsilon \\
\delta H_{\mu\nu\lambda} &= 3i\bar{\epsilon}\Gamma_{[\mu\nu}\partial_{\lambda]}\Psi
\end{aligned}$$

with

$$\Gamma_{012345}\epsilon = \epsilon \qquad \text{and} \qquad \Gamma_{012345}\Psi = -\Psi$$

This algebra closes on-shell up to translations, with e.o.m.

$$\partial_\mu\partial^\mu X^I = \Gamma^\mu\partial_\mu\Psi = \partial_{[\mu}H_{\nu\lambda\rho]} = 0$$

Make this ‘nonabelian’: Assume fields take values in some vector space with basis T^A such that $X^I = X^I_A T^A$

Promote the derivatives to covariant derivatives

$$D_\mu X^I_A = \partial_\mu X^I_A - \tilde{A}_\mu{}^B{}_A X^I_B$$

with $\tilde{A}_\mu{}^B{}_A$ is a new gauge field.

Propose a nonabelian ansatz analogous to that of the M2-brane

Consider:

$$\begin{aligned}\delta X^I &= i\bar{\epsilon}\Gamma^I\Psi \\ \delta\Psi &= \Gamma^\mu\Gamma^I\partial_\mu X^I\epsilon + \frac{1}{3!}\frac{1}{2}\Gamma_{\mu\nu\lambda}H^{\mu\nu\lambda}\epsilon\end{aligned}$$

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\begin{aligned}
\delta X^I_A &= i\bar{\epsilon}\Gamma^I\Psi_A \\
\delta\Psi_A &= \Gamma^\mu\Gamma^I D_\mu X^I_A\epsilon + \frac{1}{3!}\frac{1}{2}\Gamma_{\mu\nu\lambda}H^{\mu\nu\lambda}_A\epsilon \\
&\qquad -\frac{1}{2}\Gamma_\lambda\Gamma^{IJ}C^\lambda_B X^I_C X^J_D f^{CDB}{}_A\epsilon \\
\delta H_{\mu\nu\lambda\ A} &= 3i\bar{\epsilon}\Gamma_{[\mu\nu}D_{\lambda]}\Psi_A + i\bar{\epsilon}\Gamma^I\Gamma_{\mu\nu\lambda\kappa}C^\kappa_B X^I_C\Psi_D g^{CDB}{}_A \\
\delta\tilde{A}_{\mu}{}^{B}{}_{A} &= i\bar{\epsilon}\Gamma_{\mu\lambda}C^\lambda_C\Psi_D h^{CDB}{}_A \\
\delta C^\mu_A &= 0
\end{aligned}
$$

Here $f^{CDB}{}_A$, $g^{CDB}{}_A$ and $h^{CDB}{}_A$ are ‘structure’ constants with properties to be determined.

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\delta\tilde{A}_{\mu\ A}^{\ B} &= i\bar{\epsilon}\Gamma_{\mu\lambda}C_C^\lambda\Psi_D h^{CDB}{}_A \\
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\end{aligned}
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Here $f^{CDB}{}_A$, $g^{CDB}{}_A$ and $h^{CDB}{}_A$ are some objects with properties to be determined.

Consistency of these transformations with respect to their scaling dimensions gives

$$[H] = [X] + 1\ , \qquad [\tilde{A}] = 1\ , \qquad [C] = 1 - [X]$$

$$[\epsilon] = -\tfrac{1}{2}\ , \qquad [\Psi] = [X] + \tfrac{1}{2}\ , \qquad [X]$$

The assignments are all related to the choice of $[X]$. For the canonical choice $[X] = 2$ we have that $[C] = -1$.

Susy closure

We find that the susy algebra closes on-shell up to a translation and a gauge transformation, subject to the constraints:

$$g^{ABC}{}_{D} = h^{ABC}{}_{D} = f^{ABC}{}_{D} = f^{[ABC]}{}_{D}$$

and

$$f^{[ABC}{}_{E}f^{D]EF}{}_{G} = 0$$

This is the fundamental identity for real 3-algebras (the $\mathcal{N} = 8$ 3-algebras in 3d theories).

E.o.m. for X^I_A:

$$D^2 X^I = \frac{i}{2}\bar{\Psi}_C C^\nu_B \Gamma_\nu \Gamma^I \Psi_D f^{CDB}{}_A + C^\nu_B C_{\nu G} X^J_C X^J_E X^I_F f^{EFG}{}_D f^{CDB}{}_A$$

E.o.m. for Ψ_A:

$$\Gamma^\mu D_\mu \Psi_A + X^I_C C^\nu_B \Gamma_\nu \Gamma^I \Psi_D f^{CDB}{}_A = 0$$

E.o.m. for $H_{\mu\nu\lambda\ A}$:

$$D_{[\mu} H_{\nu\lambda\rho]\ A} = -\frac{1}{4}\epsilon_{\mu\nu\lambda\rho\sigma\tau} C^\sigma_B f^{CDB}{}_A \left(X^I_C D^\tau X^I_D + \frac{i}{2}\bar{\Psi}_C \Gamma^\tau \Psi_D \right)$$

E.o.m for $\tilde{A}_{\mu}{}^{A}{}_{B}$:

$$\tilde{F}_{\mu\nu}{}^{B}{}_{A} = C^{\lambda}_{C} H_{\mu\nu\lambda\, D} f^{BDC}{}_{A}$$

$\Rightarrow$ No new d.o.f. are introduced on-shell.

Constraints on C^{μ}_{A}:

$$D_{\nu} C^{\mu}_{A} = 0\ , \qquad C^{\lambda}_{B} C^{\rho}_{C} f^{CDB}{}_{A} = 0$$

and

$$0 = C^{\rho}_{C} D_{\rho}\Big(X^{I}_{D}, \Psi_{D}, H_{\mu\nu\lambda\, D}\Big) f^{CDB}{}_{A}$$

The structure constants are those of a real 3-algebra. Endow it with a metric

$$h^{AB} = \mathrm{Tr}(T^A, T^B)$$

$\exists$ two kinds of real 3-algebras (depending on signature):

- The Euclidean $\mathcal{A}_4$-algebra, with $f^{ABCD} = \epsilon^{ABCD}$ [Papadopoulos, Gauntlett-Gutowski]
- The Lorentzian algebras [Gomis-Milanesi-Russo, Benvenuti-Rodríguez-Gómez-Tonni-Verlinde, Ho-Imamura-Matsuo]

Lorentzian 3-algebras: start with ordinary Lie algebra $\mathcal{G}$ and add two lightlike generators $T^{\pm}$ such that $A = +, -, a, b, ...$ The structure constants are given by

$$f^{ABC}{}_{D} \rightarrow \quad f^{+ab}{}_{c} = f^{ab}{}_{c}\,,\; f^{abc}{}_{-} = f^{abc}\,,$$

The metric is given by

$$h_{AB} = \left(\begin{array}{cc|ccc} 0 & -1 & 0 & \ldots & 0 \\ -1 & 0 & 0 & \ldots & 0 \\ \hline 0 & 0 & & & \\ \vdots & \vdots & & h_{\mathcal{G}} & \\ 0 & 0 & & & \end{array}\right) .$$

Spacelike reduction

Use the Lorentzian 3-algebra and look for vacua of the theory when $\mathcal{G} = \mathfrak{su}(N)$:

$$X^I_A \to X^I_a, X^I_\pm$$

Get two abelian $(2,0)$ tensor multiplets ($X^I_\pm, \Psi_\pm, H_{\mu\nu\lambda\,\pm}$)

Next look at nonabelian piece...

$\Rightarrow$ Expand around

$$\langle C_A^\lambda \rangle = g \delta_5^\lambda \delta_A^+$$

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  $\Rightarrow$ Physics is five-dimensional

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  $\Rightarrow$ $g^{\frac{1}{2}}$ has correct scaling dimension for g_{YM} in 5d.

Make identifications

$$g = g_{YM}^2 \ , \qquad H^a_{\alpha\beta 5} = \frac{1}{g_{YM}^2} F^a_{\alpha\beta}$$

and recover e.o.m., Bianchi identity and susy xfms of five-dimensional $\mathrm{SU}(N)$ SYM theory.

$\Rightarrow$ Lorentzian theory expanded around $\langle C^\lambda_A \rangle = g\delta^\lambda_5 \delta^+_A$ is 5d SYM along with two 6d free $(2,0)$ tensor multiplets.

The off-shell $\mathrm{SO}(5,1)$ Lorentz and conformal symmetries are spontaneously broken to $\mathrm{SO}(4,1)$ Lorentz invariance.

Very similar to what happened for Lorentzian M2-brane theories in relation to D2-branes. [Gomis-Rodríguez-Gómez-Van Raamsdonk-Verlinde, Ezhuthachan-Mukhi-CP]

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$\Rightarrow$ Euclidean $\mathcal{A}_4$-algebra not dramatically different:

A single free $(2,0)$ tensor multiplet plus $\mathrm{SU}(2)$ 5d SYM.

Null Reduction

One could also consider 6d coordinates $x^\mu = (u, v, x^i)$ where $u = \frac{1}{\sqrt{2}}(x^0 - x^5)$, $v = \frac{1}{\sqrt{2}}(x^0 + x^5)$ and $i = 1, 2, 3, 4$.

Expand around

$$\langle C^\mu_A \rangle = g\delta^\mu_v \delta^+_A$$

$\Rightarrow$ Abelian sector again consists of two 6-dimensional $(2, 0)$ tensor multiplets.

$\Rightarrow$ Nonabelian sector is a supersymmetric system in 4 space and 1 null dimensions with 16 susies and $\mathrm{SO}(5)$ R-symmetry.

Look for BPS solutions to this theory:

⇒ Abelian solutions: Right-moving modes ($D_v = 0$) of selfdual strings and their 'neutral string' generalisations [Howe-Lambert-West, Gauntlett-Lambert-West]

⇒ Nonabelian solutions: Right-moving modes of 'dyonic instanton' string with lightlike profile [Lambert-Tong]]

$$H_{uvi\ a} = D_i X_a^6 \,, \qquad H_{vij\ a} = -\frac{1}{2}\epsilon_{ijkl} H_{vkl\ a} \qquad D^i D_i X_a^6 = 0$$

Summary

- ◇ Starting from abelian M5-brane susy transformations, we constructed a nonabelian $(2,0)$ tensor multiplet
- ◇ We recovered the presence of 3-algebras in this 6d theory
- ◇ Around $\langle C^\mu_A \rangle = g\delta^\mu_5 \delta^+_A$ physics were 5d SYM plus free 6d abelian $(2,0)$ tensor multiplets
- ◇ Around $\langle C^\mu_A \rangle = g\delta^\mu_v \delta^+_A$ physics were 4 space, 1 null direction susy system plus 6d abelian $(2,0)$ tensor multiplets

- We found solutions corresponding to lightlike ‘dyonic instanton’ strings as the right-moving BPS states of M2-branes suspended between parallel M5-branes
- Although the M-theory interpretation of our $(2, 0)$ tensor multiplet is unclear, interesting to see these solutions arise
- Due to its potential connection with multiple M5-branes, this system warrants further investigation