$\mathcal{N}=2$ 4d gauge theories , Ni

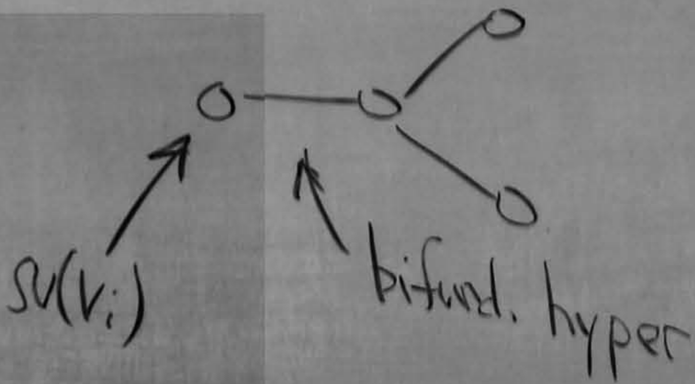
* $G = \prod SU(v_i)$ vector multiplet.

* repr: fund.
bifund.
adjoint rep

* $\beta_i \le 0$

Nekrasov, V.P.

$$-\beta_i = 2N_i - \sum_{\mathrm{bifund}(j,i)\neq 0} N_j - \overset{\#\text{ fund. to } i}{\overline{w}_i} \geq 0$$



$$C_{ij} = \begin{pmatrix} 2 & \cdots & -\#_{ij} \\ & \ddots & \\ & & 2 \end{pmatrix}$$ – Cartan matr

$\Longrightarrow$ 1) ADE classes of Cartan matrices.

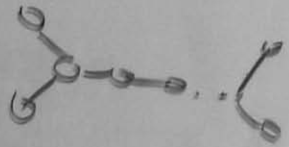
2) affine ADE

1.  A

 D

 E_6, E_7, E_8

2. $\hat{A}$ 

$\hat{D}$ 

E_6 

integration over inst. moduli $\mathcal{M}$

$$Z = \int DA\, e^{-S[A]} = \sum_{k=0}^{\infty}{}' \int_{\mathcal{M}_k} \mathrm{eu}(\ker \not{D}_R)$$

ϵ_1, ϵ_2.

$$\mathcal{M}_k = \{F_A^+ = 0\}$$

$a_{i\alpha}$ - Coulomb parameters $\alpha = 1 \ldots v_i$

$\{m\}$ - fund, bifund

$q_i = \exp(2\pi i \tau_i)$

$\mathbb{R}^4$

$\circlearrowleft SO(4)$

(ϵ_1, ϵ_2).

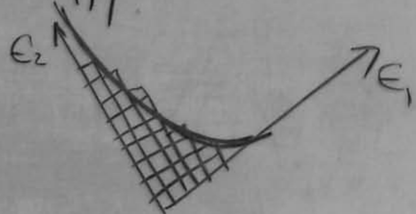
$\epsilon_1\epsilon_2 = \text{eq.}\, \mathrm{Vol}\ \mathbb{R}^4$

$$\int_{\mathbb{R}^4}^{eq(\epsilon_1\epsilon_2)} 1 = \frac{1}{\epsilon_1\epsilon_2}$$

$$Z^{(a,m,q)} = \exp\left(-\frac{1}{\epsilon_1\epsilon_2} F_{SW}(a,m,q)\right)$$

$\epsilon_1\epsilon_2 \to 0$

Nekrasov - Okounkov .



A_1 ○

$\hat{A}_0$ 

$\epsilon_1\epsilon_2 \to 0$

$\sum_{\text{part}} \to$ single dominant profile

$y(x)$ Seiberg-Witten

$$\{ [B_1, B_2] - I J = 0 \} / GL(k)$$

$k \times k$ $\quad$ $k \times N$ $\quad$ I J $\quad$ multiplet

$\{\phi_i\}$ $\quad i = 1 \ldots k$

$$\sum \delta(x - \phi_i)$$

$$\mathcal{E}_i = \sum_\alpha e^{i a_{i\alpha}} - \Big(\sum_{\text{boxes}_{\neq 1}\text{'s}} e^{i\varphi}\Big)\big(1 - e^{i\epsilon_1}\big)\big(1 - e^{i\epsilon_2}\big)$$

$$\varphi_{(i,j)} = a + (i-1)\epsilon_1 + (j-1)\epsilon_2$$

$$= \int \rho(x)\, dx$$

$$\rho(x) = \sum_i \delta(x - a_{i\alpha}) - \sum_{\text{boxes}} \delta(x-\varphi) - \delta(x-\varphi+\epsilon_1) - \delta(x-\varphi+\epsilon_2) + \delta(x-\varphi+\epsilon_1+\epsilon_2)$$

$$\int \rho_I(x)\, K_{\epsilon_1\epsilon_2}(x-x')\, C_{IJ}\, \rho_J(x')$$

Φ_I scalars of vect $su(V_i)$ class. $\Phi_I = \begin{pmatrix} a_{i1} & & \\ & \ddots & \\ & & a_{iN} \end{pmatrix}$

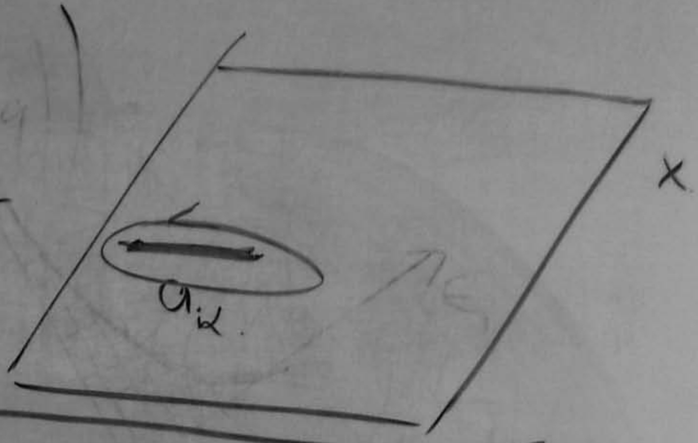
$\langle \mathrm{tr}\, \Phi_I^k \rangle =: \int \rho_I(x)\, x^k\, dx$ $\quad \sum a_{i\alpha}^k$

$$K_{\epsilon_1 \epsilon_2}^{(x)} \overset{reg}{=} \log\left(\prod_{n_1, n_2 \geq 0} (x + n_1\epsilon_1 + n_2\epsilon_2)\right) = \frac{d}{ds}\Big|_{s=0} \frac{1}{\Gamma(s)} \int \frac{e^{-xt}}{(1-e^{-\epsilon_1 t})(1-e^{-\epsilon_2 t})} t^{s-1} dt$$

$$\epsilon_1 \epsilon_2 \to 0.$$

$$K(x) = \frac{1}{\epsilon_1 \epsilon_2} \frac{x^2}{2} \left(\log x - \frac{3}{2} \right) + \dots$$

$$K''(x) = \log(x)$$



$$\boxed{\begin{array}{ll} y_i = \exp \int \rho(x) \log(x - x')\, dx' & \int_{I_{i\alpha}} \rho(x)\, dx = 1 \\ y_i(x + i0)\, y_i(x - i0) = q_i \prod_{j \neq 0} y_j(x)^{-c_{ij}} & \int_{I_{i\alpha}} \rho(x)\, x\, dx = a_{i\alpha} \end{array}}$$

$$Y_i(x - i0) = q_i \frac{1}{Y_i(x+i0)} \prod Y_j^{-C_{ij}} \iff \text{Weyl refl} \ (ADE) \ \alpha_i$$

of ADE group

$$g(x) = \prod Y_i(x)^{\alpha_i^\vee} Q_i(x)^{\Lambda_i^\vee}$$

$$\operatorname{tr}_R g(x) = (Y_i + \ldots\ldots) = P_i(x) \cdot ({}^{x}X^{V_i}$$

$R = \Lambda_i$

elliptic curve.

$$Q_i = q \prod (x - m_i)$$

$$q = \prod q_i^{a_i}$$

SW curve $\left\{ \mathcal{X}_i (g(x)) = P_i(x) \right.$ $\deg P = v_i$

$i = 1, \dots, \#$ nodes.