# Dual superconformal symmetry of scattering amplitudes in $\mathcal{N} = 4$ super-Yang-Mills

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Based on work in collaboration with

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## Plan of the talk

1. Introduction
2. Dual conformal symmetry of gluon amplitudes
3. Superamplitudes in on-shell superspace
4. Dual superconformal symmetry: MHV superamplitudes
5. Dual superconformal symmetry: non-MHV superamplitudes
6. Conclusions and outlook

# 1 Introduction

## 1.1 Scattering amplitudes in $\mathcal{N} = 4$ SYM

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Planar color-ordered $n$–particle (gluons, gluinos, scalars) scattering amplitudes are functions of light-like momenta $p_i^2 = 0$ and helicities $h_i = \pm 1, \pm 1/2, 0$ $(i = 1 \ldots n)$, given by their perturbative expansion in $a = g^2 N/8\pi^2$:

$$\mathcal{A}_n(p_i, h_i) = \mathcal{A}_{n;0} + a \sum_H \mathcal{A}_{n;1}^H \, M_{n;1}^H(p_i) + O(a^2)$$

$\mathcal{A}_{n;0} \longrightarrow$ tree amplitude depending on helicities
$\mathcal{A}_{n;1}^H \longrightarrow$ one-loop helicity structure $H$; the sum goes over all independent $H$
$M_{n;1}^H \longrightarrow$ one-loop scalar Feynman integrals
IR divergences $\Rightarrow$ dimensional regularization
♣
Simplest example: Maximally Helicity Violating (MHV) amplitudes, e.g. for gluons:
$(- - + \ldots +)$, $(- + - + \ldots +)$, etc.
Unique helicity structure (tree):

$$\mathcal{A}_n^{\rm MHV}(p_1^-, p_2^-, p_3^+, \ldots, p_n^+) = \mathcal{A}_{n;0}^{\rm MHV} \, M_n^{\rm MHV}(p_i)$$

$$M_n^{\rm MHV} = 1 + aM_n^{(1)} + O(a^2)$$

♣

$\mathcal{N} = 4$ SYM is a (super)conformal theory $\Rightarrow$ conformal symmetry of $\mathcal{A}_n(p_i)$ ?

Two problems:

**(i)** Conformal boosts realized on momenta are 2nd-order differential operators (Witten)

**(ii)** IR divergences break conformal symmetry

Can we do better?

## 1.2 Dual conformal symmetry

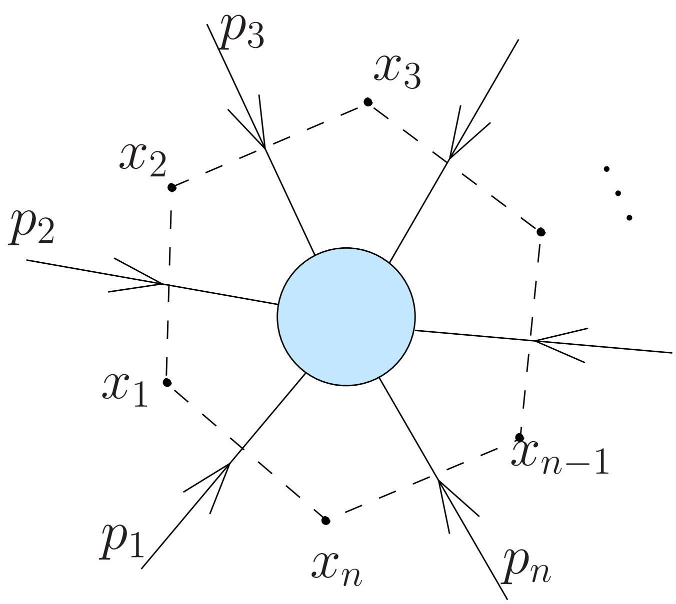
Hidden symmetry of $\mathcal{A}_n$ of dynamical origin:

- Linear action on the particle momenta in

Dual space:

$$p_i = x_i - x_{i+1} \equiv x_{i\ i+1} \quad \Leftrightarrow \quad \sum_i p_i = 0 \ \text{if} \ x_{n+1} \equiv x_1$$

Simple change of variables, not a Fourier transform!



• Usual conformal group $SO(4,2)$ acting on the dual coordinates $\longrightarrow$ dual conformal symmetry.

Conformal group = Poincaré + inversion:

$$x^\mu \longrightarrow \frac{x^\mu}{x^2} \quad : \qquad x_{ij}^2 \longrightarrow \frac{x_{ij}^2}{x_i^2 x_j^2}$$

Recall the structure of the amplitude:

$$\mathcal{A}_n(p_i, h_i) = \mathcal{A}_{n;0} + a \sum_H \mathcal{A}_{n;1}^H \, M_{n;1}^H(p_i) + O(a^2)$$

• Exact symmetry of $\mathcal{A}_{n;k}$, $k = 0, 1, \ldots$ (for split-helicity amplitudes, and for the entire superamplitude)

• Anomalous symmetry of $M_{n;k}$ controlled by WI:

Example: MHV amplitudes

$$\ln M_n^{\mathrm{MHV}} = \ln Z_n + \ln F_n + O(\epsilon)$$

$$\ln Z_n = \sum_{l \geq 1} a^l \sum_{i=1}^n \left(-x_{i-1,i+1}^2 \mu^2\right)^{l\epsilon} \left( \frac{\Gamma_{\mathrm{cusp}}^{(l)}}{(l\epsilon)^2} + \frac{\Gamma^{(l)}}{l\epsilon} \right)$$

Anomalous CWI:

$$K^\mu \ln F_n = \frac{1}{2} \Gamma_{\mathrm{cusp}}(a) \sum_{i=1}^n \ln \frac{x_{i,i+2}^2}{x_{i-1,i+1}^2} x_{i,i+1}^\nu$$

Fixes the form of $\ln F_n$ for $n = 4, 5$ but not for $n \geq 6$

♣

Main claim: Exact symmetry of the finite 'ratio' $\mathcal{R}_n$

$$\mathcal{A}_n = \mathcal{A}_n^{\rm MHV} \times [\mathcal{R}_n + O(\epsilon)]$$

- Conformal anomaly contained in MHV prefactor

♣

What about the helicity structures?

## 2 Dual conformal symmetry of gluon amplitudes

Question: Can we generalize dual conformal symmetry to non-MHV amplitudes?

$$\mathcal{A}_n(p_i, h_i) = \mathcal{A}_{n;0} + a \sum_H \mathcal{A}^H_{n;1} \, M^H_{n;1}(p_i) + O(a^2)$$

Helicity structures $\mathcal{A}_{n;0}$, $\mathcal{A}^H_{n;1}$; loop corrections $M^H_{n;1}$

Start with the simplest case of MHV amplitudes $\longrightarrow$ unique helicity structure.

## 2.1 MHV tree level

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Spinor helicity formalism: commuting spinors $\lambda^\alpha$, $\tilde{\lambda}^{\dot{\alpha}}$

$$p_i^2 = 0 \;\Leftrightarrow\; p_i^{\alpha\dot{\alpha}} \equiv p_i^\mu (\sigma_\mu)^{\alpha\dot{\alpha}} = \lambda_i^\alpha \, \tilde{\lambda}_i^{\dot{\alpha}}$$

$$\mathcal{A}_{n;0}^{\rm MHV}(\ldots i^- \ldots j^- \ldots) = \delta^{(4)}(\sum_{k=1}^{n} p_k) \, \frac{\langle i\, j\rangle^4}{\langle 1\, 2\rangle\langle 2\, 3\rangle \ldots \langle n\, 1\rangle}$$

Lorentz invariant spinor contractions

$$\langle i\, j\rangle = -\langle j\, i\rangle = \epsilon^{\alpha\beta}\lambda_{i\,\alpha}\,\lambda_{j\,\beta}$$

carrying helicities $-1/2$ at points $i$ and $j$

♣

Is it dual conformal?

## 2.2 Dual conformal transformations of spinors

♣

Dual coordinates $\longrightarrow$ spinor variables:

$$p_i^{\alpha\dot\alpha} = (x_i - x_{i+1})^{\alpha\dot\alpha} = \lambda_i^\alpha\,\tilde\lambda_i^{\dot\alpha} \qquad \Rightarrow\ \lambda_i^\alpha\,(x_i - x_{i+1})_{\alpha\dot\alpha} = 0$$

♣

Conformal inversion in dual space:

$$I[x_i - x_j] = x_i^{-1}(x_i - x_j)x_j^{-1}\ \Rightarrow$$

$$\begin{aligned} I[x_i - x_{i+1}] &= x_i^{-1}(\lambda_i\,\tilde\lambda_i)x_{i+1}^{-1}\ \Rightarrow \\ I[\lambda_i^\alpha] &= \frac{\lambda_i^\alpha(x_i)_{\alpha\dot\alpha}}{x_i^2} \equiv \lambda_i\,x_i^{-1} \\ &= \lambda_i^\alpha\frac{(x_{i+1})_{\alpha\dot\alpha}}{x_i^2} \end{aligned}$$

♣

Conformal properties of $\langle i\,j\rangle$:

$$I[\langle i\ i+1\rangle] = \langle i|\frac{x_{i+1}}{x_i^2}x_{i+1}^{-1}|i+1\rangle = \frac{\langle i\ i+1\rangle}{x_i^2}$$

$\langle i\ i+1\rangle$ is dual conformal, but not $\langle i\,j\rangle$ for $j \neq i+1$ !

♣

The rational factor in $\mathcal{A}_{n;0}^{\rm MHV}$ is dual covariant only if the negative-helicity gluons are adjacent (‘split-helicity’ amplitudes).

## 2.3 Properties of the delta function

$\delta^{(4)}(\sum_{i=1}^{n} p_i)$ imposes momentum conservation:

$$\sum_{i=1}^{n} p_i = 0 \;\Leftrightarrow\; \sum_{i=1}^{n} (x_i - x_{i+1}) = 0 \;\text{ iff }\; x_{n+1} \equiv x_1$$

$\longrightarrow$ cyclic symmetry

♣
Relax cyclicity, $x_1 \neq x_{n+1}$, and then impose it by

$$\delta^{(4)}(x_1 - x_{n+1}) \;\longrightarrow\; \text{manifestly dual conformal}$$

## 2.4 Split-helicity non-MHV tree amplitudes

♣
Split-helicity MHV tree amplitudes are dual conformal, e.g.

$$\mathcal{A}_n^{\mathrm{MHV}}(--+\ldots+) = \delta^{(4)}(x_1 - x_{n+1}) \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle \ldots \langle n\,1\rangle}$$

♣
All split-helicity non-MHV tree amplitudes are dual conformal. Checked directly using the recursion relations of Britto, Cachazo, Feng, Roiban, Spradlin, Volovich, Witten

♣

Non-split-helicity amplitudes are not dual conformal

♣

Accidental property of split-helicity amplitudes?

No, general property!

To see it, we need dual supersymmetry.

## 3 Superamplitudes in on-shell superspace

♣

Superamplitudes: compact form of all $\mathcal{N} = 4$ SYM amplitudes (gluons, gluinos and scalars) in dual superspace.

♣

Way to make dual (super)conformal symmetry manifest

### 3.1 Nair's formulation of MHV amplitudes

♣

Nair's superspace description of tree MHV amplitudes

$$\mathcal{A}_n^{\mathrm{MHV}} = \frac{\delta^{(4)}(\sum_{i=1}^n p_i)\, \delta^{(8)}(\sum_{j=1}^n \lambda_{j\,\alpha}\, \eta_j^A)}{\langle 1\, 2\rangle \langle 2\, 3\rangle \ldots \langle n\, 1\rangle}$$

$\eta_i^A$ ($A = 1 \ldots 4$ – $SU(4)$ index), with helicity 1/2, are Grassmann variables of on-shell superspace

♣
$\mathcal{N}=4$ gluon supermultiplet $\rightarrow$ PCT self-conjugate $\rightarrow$ holomorphic (chiral) description

$$\Phi(p,\eta) = G^+(p) + \eta^A\Gamma_A(p) + \eta^A\eta^B S_{AB}(p) + \eta^A\eta^B\eta^C\epsilon_{ABCD}\bar{\Gamma}^D(p) + \eta^A\eta^B\eta^C\eta^D\epsilon_{ABCD}G^-(p)$$

Particle wave functions:
- $G^\pm$ – gluons (helicity $\pm 1$);
- $\Gamma_A$, $\bar{\Gamma}^A$ – gluinos (helicity $\pm 1/2$);
- $S_{AB}$ – scalars (helicity 0)

♣
Extract, e.g., the gluon component $(--+\ldots+)$: collect $\eta^4$ terms at negative-helicity sites

$$\delta^{(8)}\left(\sum_{i=1}^{n} \lambda_{i\,\alpha}\,\eta_i^A\right) \rightarrow \langle 12\rangle^4\ \eta_1^4\eta_2^4\eta_3^0\ldots\eta_n^0$$

## 3.2 On-shell $\mathcal{N}=4$ supersymmetry

♣
Clifford algebra for massless Poincaré states:

$$q^A = \eta^A\,, \qquad \bar{q}_A = \frac{\partial}{\partial \eta^A}\,, \qquad \{q^A, \bar{q}_B\} = \delta^A_B$$

♣
Covariant description with the help of $\lambda_\alpha$:

$$q^A_\alpha = \lambda_\alpha \eta^A\,, \qquad \bar{q}_{A\,\dot\alpha} = \tilde\lambda_{\dot\alpha} \frac{\partial}{\partial \eta^A}$$

On-shell $\mathcal{N}=4$ supersymmetry ($p^2=0$):

$$\{q^A_\alpha, \bar{q}_{B\,\dot\alpha}\} = \delta^A_B\,\lambda_\alpha \tilde\lambda_{\dot\alpha} = \delta^A_B\, p_{\alpha\dot\alpha}$$

## 3.3 General superamplitudes

♣

- Translation invariance

$$p = \sum_{i=1}^{n} p_i \;\Rightarrow\; \delta^{(4)}(\sum_{i=1}^{n} p_i) = \delta^{(4)}(\sum_{i=1}^{n} \lambda_i\,\tilde\lambda_i)$$

- On-shell $q$–supersymmetry

$$q^A_\alpha = \sum_{i=1}^{n} (q_i)^A_\alpha \;\Rightarrow\; \delta^{(8)}(\sum_{i=1}^{n} \lambda_{i\,\alpha}\,\eta_i^A)$$

♣
General superamplitude

$$\mathcal{A}_n(\lambda,\tilde\lambda,\eta) \;=\; \delta^{(4)}(\sum_{i=1}^{n}\;\lambda_i\,\tilde\lambda_i)\;\delta^{(8)}(\sum_{j=1}^{n}\;\lambda_j\,\eta_j)$$
$$\times\;\left[\mathcal{A}_n^{(0)}+\mathcal{A}_n^{(4)}+\ldots+\mathcal{A}_n^{(4n-16)}\right]$$

$\mathcal{A}_n^{(4k)}(\eta)$ – homogeneous polynomials of degree $4k$:

$$\begin{aligned} k=0 \;&\rightarrow\; \text{MHV} \\ k=1 \;&\rightarrow\; \text{Next-to-MHV} \\ \ldots \\ k=n-4 \;&\rightarrow\; \overline{\text{MHV}} \end{aligned}$$

♣
Simplest case – MHV:

$$\mathcal{A}_n^{(0)}=\frac{1}{\langle 1\,2\rangle\langle 2\,3\rangle\ldots\langle n\,1\rangle}\;M_n(p)$$

Complete all-order MHV superamplitude:

$$\mathcal{A}_n^{\rm MHV}(\lambda,\tilde\lambda,\eta)=\frac{\delta^{(4)}(\sum_{i=1}^n\;p_i)\;\delta^{(8)}(\sum_{j=1}^n\;\lambda_{j\,\alpha}\,\eta_j^A)}{\langle 1\,2\rangle\langle 2\,3\rangle\ldots\langle n\,1\rangle}\,M_n(p)$$

♣
Rewrite the general superamplitude by pulling out MHV:

$$\mathcal{A}_n = \mathcal{A}_n^{\mathrm{MHV}} \left[ 1 + \mathcal{P}_n^{(4)} + \ldots + \mathcal{P}_n^{(4n-16)} + O(\epsilon) \right]$$

$\mathcal{P}_n^{(4k)}$ are finite and nilpotent. They contain helicity structures and loop corrections for all non-MHV superamplitudes.

♣
Conjecture: all $\mathcal{P}_n^{(4k)}$ are exactly dual superconformal. The dual conformal anomaly is in the IR divergent MHV prefactor.

# 4 Dual superconformal symmetry I: MHV superamplitudes

## 4.1 Dual superspace

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Introduce dual superspace coordinates:

$$\sum_{i=1}^{n} p_i = 0 \ \rightarrow \qquad p_i = x_i - x_{i+1}\,, \quad x_{n+1} = x_1$$

$$\sum_{i=1}^{n} \lambda_i \eta_i = 0 \ \rightarrow \qquad \lambda_{i\,\alpha}\, \eta_i^A = (\theta_i - \theta_{i+1})_\alpha^A\,, \quad \theta_{n+1} = \theta_1$$

♣
Dual chiral superspace

$$(x_{\alpha\dot{\alpha}}\,,\,\theta^A_\alpha\,,\,\lambda_\alpha)$$

Defining constraints:

$$\lambda^\alpha_i\,(x_i - x_{i+1})_{\alpha\dot{\alpha}} = 0 \;\longrightarrow\; \text{derive } \tilde{\lambda}^{\dot{\alpha}}_i$$
$$\lambda^\alpha_i\;(\theta_i - \theta_{i+1})^A_\alpha = 0 \;\longrightarrow\; \text{derive } \eta^A_i$$

## 4.2 Dual $\mathcal{N} = 4$ superconformal symmetry

♣
$\mathcal{N} = 4$ super-Poincaré algebra in dual superspace

$$Q_{A\,\alpha} = \sum_{i=1}^{n} \frac{\partial}{\partial \theta_i^{A\,\alpha}}, \quad \bar{Q}^A_{\dot{\alpha}} = \sum_{i=1}^{n} \theta_i^{A\,\alpha} \frac{\partial}{\partial x_i^{\dot{\alpha}\alpha}}, \; P_{\alpha\dot{\alpha}} = \sum_{i=1}^{n} \frac{\partial}{\partial x_i^{\dot{\alpha}\alpha}}$$

$$\{Q_{A\,\alpha}, \bar{Q}^B_{\dot{\alpha}}\} = \delta^B_A\; P_{\alpha\dot{\alpha}}$$

♣
Conformal inversion for dual superspace coordinates

$$I[x_i] = x_i^{-1}\,, \quad I[\theta_i] = \theta_i x_i^{-1}\,, \quad I[\lambda_i] = \lambda_i x_i^{-1}$$

♣
From Poincaré to conformal supersymmetry:
$\longrightarrow$ Conformal boosts: $K = IPI$
$\longrightarrow$ Special conformal supersymmetry : $(S, \bar{S}) = I(Q, \bar{Q})I$
$\longrightarrow$ Central charge = helicity !

### 4.3 Dual superconformal symmetry of MHV superamplitudes

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Properties of the delta functions:

Relax cyclicity, $x_{n+1} \neq x_1$, $\theta_{n+1} \neq \theta_1$, and impose it through delta function. Then, only in $\mathcal{N} = 4$,

$$
\begin{aligned}
I[\delta^{(4)}(x_1 - x_{n+1})] &\rightarrow x_1^8\, \delta^{(4)}(x_1 - x_{n+1}) \\
I[\delta^{(8)}(\theta_1 - \theta_{n+1})] &\rightarrow x_1^{-8}\, \delta^{(8)}(\theta_1 - \theta_{n+1})
\end{aligned}
$$

♣

MHV superamplitude in dual superspace

$$
\mathcal{A}_n^{\rm MHV}(x, \theta, \lambda) = \frac{\delta^{(4)}(x_1 - x_{n+1})\, \delta^{(8)}(\theta_1 - \theta_{n+1})}{\langle 1\, 2\rangle \langle 2\, 3\rangle \ldots \langle n\, 1\rangle}\, M_n(x_{ij})
$$

Tree – manifestly dual (super)conformal covariant.

Loops – IR divergent factor $M_n(x_{ij})$ satisfies anomalous dual conformal Ward identity

♣

Part of the superconformal algebra $(Q, \bar{S}, P)$ is a symmetry of the whole amplitude, and $(\bar{Q}, S, K, D)$ only of the helicity structures (due to anomalies)

# 5 Dual superconformal symmetry II: non-MHV superamplitudes

## 5.1 Conjecture

Recall the general structure of the superamplitude

$$\mathcal{A}_n = \mathcal{A}_n^{\rm MHV}(a,\epsilon)\left[1 + \mathcal{P}_n^{(4)} + \ldots + \mathcal{P}_n^{(4n-16)} + O(\epsilon)\right]$$

♣ $\mathcal{A}_n^{\rm MHV}$ is the full MHV amplitude, containing the IR divergences and satisfying an anomalous dual CWI $\Leftrightarrow$ Wilson loop

♣ $\mathcal{P}_n^{(4)}$ are finite dual superconformal nilpotent invariants

## 5.2 Evidence: one-loop NMHV superamplitudes

The complete one-loop NMHV superamplitude, whose gluon part was found by Bern, Dixon, Kosower, is described by the dual superconformal invariant

$$\mathcal{P}_n^{(4)} = \sum_{p,q,r=1}^{n} c_{pqr}\ \delta^{(4)}(\Xi_{pqr})\ M_{pqr}(x_{ij})$$

♣

$$\begin{aligned}\Xi_{pqr} &= \langle p|\left[x_{pq}x_{qr}(|\theta_r\rangle - |\theta_p\rangle) + x_{pr}x_{rq}(|\theta_q\rangle - |\theta_p\rangle)\right] \\ &= -\langle p|\left(x_{pq}x_{qr}\sum_{i=p}^{r-1}|i\rangle\eta_i + x_{pr}x_{rq}\sum_{i=p}^{q-1}|i\rangle\eta_i\right)\end{aligned}$$

is a 3-point dual superconformal covariant of degree 4

♣

$$c_{pqr} = \frac{\langle q-1\, q\rangle\langle r-1\, r\rangle}{x_{qr}^2\langle p|x_{pr}x_{r\,q-1}|q-1\rangle\langle p|x_{pr}x_{r\,q}|q\rangle\langle p|x_{pq}x_{q\,r-1}|r-1\rangle\langle p|x_{pq}x_{q\,r}|r\rangle}$$

is a dual conformal covariant

♣

$$c_{pqr}\ \delta^{(4)}(\Xi_{pqr})$$

is a 3-point dual superconformal invariant of degree 4

♣

$$M_{pqr}(x_{ij}) = 1 + aM_{pqr}^{(\text{one-loop})} + \ ?\ O(a^2)$$

are dual conformal invariant functions, made of finite combinations of one-loop scalar box integrals

## 5.3 Comments

♣
The superstructure

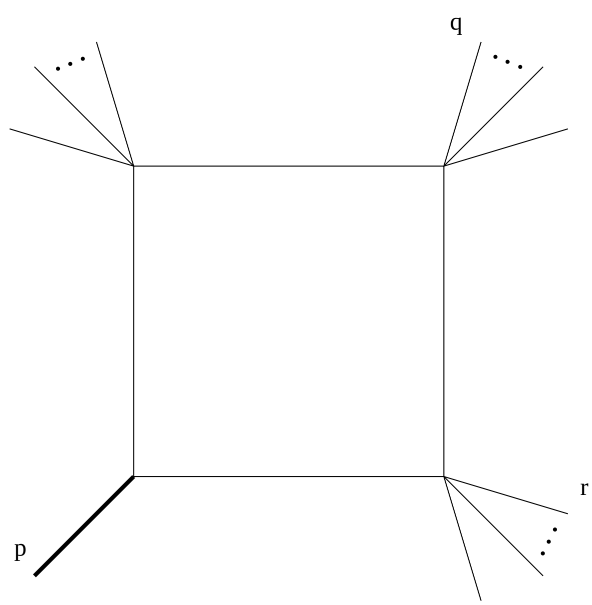
$$\delta^{(8)}(\sum_{i=1}^{n} \lambda_{i\,\alpha}\,\eta_i^A)\ c_{pqr}\ \delta^{(4)}(\Xi_{pqr})$$

encodes all helicity structures for gluons, gluinos, scalars.
In particular

$$\mathcal{H}_{m_1 m_2 m_3}\ \eta_{m_1}^4\ \eta_{m_2}^4\ \eta_{m_2}^4$$

describes gluon NMHV amplitudes with negative-helicity gluons at sites $m_1, m_2, m_3$.

$\mathcal{H}_{m_1 m_2 m_3}$ $\Leftrightarrow$ 3-mass-box coefficients of Bern, Dixon, Kosower

$\Xi_{pqr}$ $\Leftrightarrow$ 

♣

Expanding in $\eta_i$ breaks manifest dual conformal symmetry, except for split-helicity terms. The non-split-helicity ones transform into each other

♣

An early result for $n = 6$ NMHV in a paper by Huang.

### 5.4 NMHV tree-level superamplitudes

As a byproduct, we get a new, manifestly Lorentz covariant form of the NMHV tree superamplitude

$$\mathcal{A}_{n;0}^{\mathrm{NMHV}} = \delta^{(4)}(\sum_{i=1}^{n} \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_{j=1}^{n} \lambda_j\, \eta_j) \sum_{p,q,r=1}^{n} c_{pqr}\, \delta^{(4)}(\Xi_{pqr})$$

Compare to the MHV × MHV construction of Cachazo, Svrcek, Witten, or to its supersymmetric version by Georgio, Glover, Khoze, who need a reference spinor:

$$\mathcal{A}_{n;0}^{\rm NMHV} = \delta^{(4)}(\sum_{i=1}^{n} \lambda_i\tilde{\lambda}_i)\, \delta^{(8)}(\sum_{j=1}^{n} \lambda_j\eta_j)$$
$$\times \Big[\sum_{q,r} \frac{\delta^{(4)}(\sum_{k=q}^{r-1}\langle I_r\, k\rangle\eta_k + \sum_{k=1}^{q-1}(\langle I_r\, k\rangle - \langle I_q\, k\rangle)\eta_k)}{x_{qr}^2\, \langle 1\, 2\rangle \ldots \langle q-1\, I_q\rangle\langle I_q\, q\rangle \ldots \langle r-1\, I_r\rangle\langle I_r\, r\rangle \ldots \langle n\, 1\rangle}$$
$$+ \ \text{cycle}\Big]$$

where

$$\langle I_q| = \langle 1|x_{1r}x_{qr}\,, \qquad \langle I_r| = \langle 1|x_{1q}x_{qr}$$

$$?\ \Longleftrightarrow\ ?$$

$$\mathcal{A}_{n;0}^{\rm CSW-GGK} = \delta^{(4)}(\sum_{i=1}^{n} \lambda_i\tilde{\lambda}_i)\, \delta^{(8)}(\sum_{j=1}^{n} \lambda_j\eta_j)$$
$$\times \sum_{q,r} \frac{\delta^{(4)}(\sum_{k=q}^{r-1}\langle I\, k\rangle\eta_k)}{x_{qr}^2\, \langle 1\, 2\rangle \ldots \langle q-1\, I\rangle\langle I\, q\rangle \ldots \langle r-1\, I\rangle\langle I\, r\rangle \ldots \langle n\, 1\rangle}$$

where

$$\langle I| = [\xi_{\rm ref}|x_{qr} \ : \qquad [\xi_{\rm ref}| \neq \langle 1|x_{1r} \neq \langle 1|x_{1q}$$

Fixed reference spinor $[\xi_{\rm ref}|$ $\Rightarrow$ breaks Lorentz !

Looks as if we were using two 'reference spinors'?

Why do the two forms of the tree coincide ???

## 6 Conclusions and outlook

♣

Dual (super)conformal symmetry is a universal feature of $\mathcal{N} = 4$ scattering amplitudes

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Its origin is unknown (dynamical). Indications from string theory by Berkovits, Maldacena.

♣

What fixes the form of the super-helicity structures

$$c_{pqr}\ \delta^{(4)}(\Xi_{pqr})\ ?$$

Dual superconformal symmetry does, if we assume 3-point invariants $\Leftrightarrow$ 3-mass-boxes.

♣

NNMHV involve 4-mass-boxes $\Rightarrow$ 4-point invariants? Need further constraints (dynamical symmetries?)

♣

Probably the "tip of an iceberg" of an (infinite?) set of symmetries $\longrightarrow$ integrability?

♣

non-MHV amplitudes provide us with finite exactly dual conformal functions. Can we find differential equations for them? $\longrightarrow$ integrability?

♣

Can the Wilson loop/string see helicity?