Numerical Unitary Formalism in D-dimension

Workshop on Gauge Theory and String Theory

Zurich, July 2-4

Fully automated algorithm of polynomial complexity for evaluating one loop amplitudes in Quantum Field Theory

R. K. Ellis, W. Giele, Z.K, arXiv.0708.2398 (EGK)

W. Giele, Z.K., K. Melnikov,arXiv0801.2237 (GKM)

R.K. Ellis, W. Giele, Z. K. in Bern et.al. (LesHouches) arXiv:0803.0494

R.K.Ellis, W. Giele, Z. K., K. Melnikov, arXiv:0806.3467 (EGKM)

Multi-leg processes in perturbation theory,standard approach

Born amplitudes

- **First estimates: MC's based on Feynman diagrams**
- **Inefficient for multi-legs:**
  Stronger than factorial growth in number of external particles
  N-gluon scattering: CPU grows as $N^{(N-3)}$ (E-algorithm)
- **Use of recursion relations** (Berends, Giele; Britto, Cachazo, Feng) **:**
  CPU time has polynomial growth in the number of the external legs
  N^{α} (P- algorithm)
- **Tree-level general purpose codes: ALPGEN, HELAC, MADGRAPH, WHIZARD, CompHep, etc..**
  Used by the experimentalists and model builders.
  MSSM, Little Higgs… etc. are implemented

Amplitudes in Next-to-Leading Order

- **Most of the results (up to 5 legs) are obtained with E-algorithms**
- **Standard Feynman-graph based method using computer codes like**
  e.g: QGRAF , FORM , Tensor Reduction (Passarino,Veltman)
  - **- Many diagrams (E-algorithm),**
  - **- Many terms from tensor reduction (new additional E-algorithm)**
  - **+ but: very systematic with much experience and analytic treatments**
- **Semi-analytic, standard methods with numerical tensor reduction pushed to their limits:** K. Ellis, Giele, Zanderighi **6g one-loop amplitudes**
- **Completely numerical methods use Feynman diagram no** tensor reduction
  Nagy, Soper; Lazapoulos, Melnikov, Petriello; Anastasiou, Beerli, Daleo
- **Present state of art**
  Denner Dittmaier **$e^+e^- \to \mu^+\mu^- \tau^+\tau^-$ QED NLO**
  Dittmaier, Uwer, Weinzierl **$p+p \to t+t$ jet NLO**

The Unitarity Method: successful P-algorithm at NLO

S-matrix theory : **two particle scattering amplitude is given in terms of its imaginary part** (Landau, ...1950's)

Perturbative gauge theories at NLO (Bern, Dixon, Dunbar, Kosower,1994) **:**

i) Decomposition of one-loop amplitudes in terms of finite number of well defined scalar integral functions
*(*Passarino, Veltman *)*

ii) imaginary part of one-loop amplitudes is given in terms of products gauge invariant tree amplitudes

Bern, Dixon, Kosower **(**1994-1998) : **construct gauge theory one-loop amplitudes from tree amplitudes.**

Early attempts

RESULTS:

i) BDK theorem: SUSY gauge theories have no rational parts
important applications to N=1,N=4 SYM

ii) Impressive QCD result: $e^+ + e^-$ annihilation to four jets in NLO (1998)
First time triple cut (generalized unitarity) is used

(analytic result, only four-dimensional states on cut lines, spinor helicity formalism, set of magic tricks, rational part is fixed from soft and collinear limits, triple cuts, SUSY identities etc.)

DIFFICULTIES:

i) Reduction of cut tensor integrals (Passarino-Veltman,…)
ii) The cut lines are treated in four dimensions (no rational parts)
iii) Only double cuts have been applied.

LIMITED DEVELOPMENT (1998-2003), even though ideas for progress have been there

i) D-dimensional integrals can be reconstructed fully from the imaginary part (van Neerven, 1986)
ii) To get the rational part treat the cut lines in D-dimension (Bern,Morgan,1996)
iii) Applications with massive fermions in the loop (Bern,Morgan 1996)

Towards Systematic treatment with general validity

INSPIRATION FROM TWISTOR FORMULATION (Witten, 2003, Santa Barbara Workshop 2004)

i) Generalized unitarity (Britto, Cachazo, Feng) , complex four momenta
ii) New tree level recursion relations (Britto,Cachazo, Feng, Witten)
iii) New loop level recursion relations for rational parts (Bern, Dixon, Koswer, …)
iv) Reduction with spinor integration (Britto, Cachazo, Feng, Mastrolia).

NEW REDUCTION METHODS

i) algebraic reduction (parametric integration) (Ossola, Papadopoulos, Pittau ,2006)
ii) D-dimensional reduction with spinor integration (scalars in the loop)
(Anastasiou, Britto, Feng, Kunszt, Mastrolia,2006)

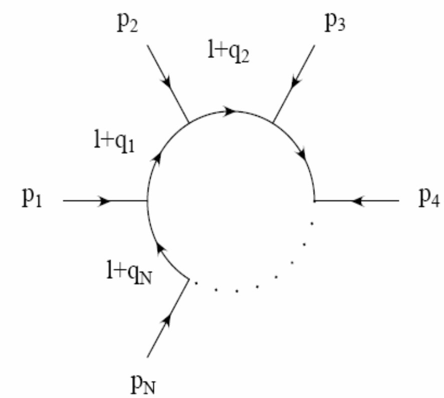
NUMERICAL IMPLEMENTATIONS (Ossola, Papadopoulos, Pittau ,2006)

i) Feynman diagram based unitarity cut method (Ossola, Papadopoulos, Pittau ,2007)
ii) Numerical implementation of unitarity method for cut-constructible part of the 6gluon amplitude (EGK, 2007)

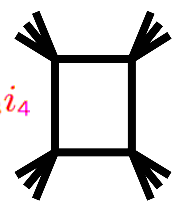
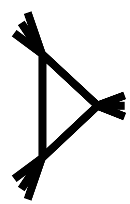
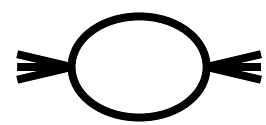
NV basis for decomposing loop momenta, no reference to one loop Feynman diagrams

Decomposing one-loop N-point amplitudes in terms of master integrals

$$\mathcal{A}_N(p_1,p_2,\ldots,p_N) = \sum_{1\le i_1<i_2<i_3<i_4\le N} d_{i_1 i_2 i_3 i_4}(p_1,p_2,\ldots,p_N) I_{i_1 i_2 i_3 i_4}$$
$$+ \sum_{1\le i_1<i_2<i_3\le N} c_{i_1 i_2 i_3}(p_1,p_2,\ldots,p_N) I_{i_1 i_2 i_3}$$
$$+ \sum_{1\le i_1<i_2\le N} b_{i_1 i_2}(p_1,p_2,\ldots,p_N) I_{i_1 i_2}$$
$$+ \sum_{1\le i_1\le N} a_{i_1}(p_1,p_2,\ldots,p_N) I_{i_1}$$



$$I_{i_1\cdots i_M} = \int [d\,l]\, \frac{1}{d_{i_1}\cdots d_{i_M}}$$

$$\mathcal{A}_N(\{p_i\}) = \sum d_{i_1 i_2 i_3 i_4}$$  $$+\sum c_{i_1 i_2 i_3}$$  $$+\sum b_{i_1 i_2}$$ 

FDHS scheme: coefficients d and c are independent from ε

Rational part is generated by the order ε part of b_{ij}.

No tadpoles for massless case. No rational parts for supersymmetric theories.

The OPP reduction using NV basis

systematic algebraic reduction at the integrand level

$$\mathcal{A}_N(p_1, p_2, \ldots, p_N; l) = \frac{\mathcal{N}(p_1, p_2, \ldots, p_N; l)}{d_1 d_2 \cdots d_N} =$$

$$\sum_{1\le i_1<i_2<i_3<i_4\le N} \frac{\bar{d}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{1\le i_1<i_2<i_3\le N} \frac{\bar{c}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{1\le i_1<i_2\le N} \frac{\bar{b}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{1\le i_1\le N} \frac{\bar{a}_{i_1}(l)}{d_{i_1}}$$

Box residues 2, triangle residues 7, bubble residues 9 structures:

$$\bar{d}_{ijkl}(l) \equiv \bar{d}_{ijkl}(n_1 \cdot l)\,, \quad (n_1 \cdot l)^2 \sim n_1^2 = 1\,, \quad \bar{d}_{ijkl}(l) = d_{ijkl} + \tilde{d}_{ijkl}\, l \cdot n_1$$

$$\bar{c}_{ijk}(l) = c_{ijk}^{(0)} + c_{ijk}^{(1)} s_1 + c_{ijk}^{(2)} s_2 + c_{ijk}^{(3)} (s_1^2 - s_2^2) + s_1 s_2 (c_{ijk}^{(4)} + c_{ijk}^{(5)} s_1 + c_{ijk}^{(6)} s_2)$$

$$\bar{b}_{ij}(l) = b_{ij}^{(0)} + b_{ij}^{(1)} s_1 + b_{ij}^{(2)} s_2 + b_{ij}^{(3)} s_3 + \ldots$$

18 l-independent parameters, vanishing (spurious) integrals

Parameterization of the loop momentum

The loop momenta can be decomposed in terms of suitable fixed basis vectors.

we use: **dual momenta v_i** $p_i v_j = \delta_{ij}$
and orthogonal **unit vectors n_i**

Decomposition of the loop-momentum

$$l^\mu = V_R^\mu + \sum_{i=1}^{D_P} \frac{1}{2}(d_i - d_{i-1})\, v_i^\mu + \sum_{i=1}^{D_T} \alpha_i\, n_i^\mu \; ,$$

$$V_R^\mu = -\frac{1}{2} \sum_{i=1}^{D_P} \left((q_i^2 - m_i^2) - (q_{i-1}^2 - m_{i-1}^2) \right) v_i^\mu$$

Solving the unitarity conditions

Contributions with four cut propagators $d_i=d_j=d_k=d_l=0$ two solutions

$$l^\mu = V_4^\mu + \alpha_1\, n_1^\mu$$

$$l_\pm^\mu = V_4^\mu \pm i\,\sqrt{V_4^2 - m_l^2} \times n_1^\mu$$

Complex valued loop momenta

Triangle, infinite # of solutions (on a circle)

$$l^\mu = V_3^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu$$

$$l_{\alpha_1\alpha_2}^\mu = V_3^\mu + \alpha_1\, n_1^\mu + \alpha_2\, n_2^\mu;\;\; \alpha_1^2 + \alpha_2^2 = -(V_3^2 - m_k^2)$$

Bubble, infinite # of solutions (on a “sphere”)

$$l^\mu = V_2^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu + \alpha_3 n_3^\mu$$

$$l_{\alpha_1\alpha_2\alpha_3}^\mu = V_2^\mu + \alpha_1\, n_1^\mu + \alpha_2\, n_2^\mu + \alpha_3\, n_3^\mu;\;\; \alpha_1^2 + \alpha_2^2 + \alpha_3^2 = -(V_2^2 - m_j^2)\ .$$

The parameters are obtained by algebraic equations

The residue is taken at special loop momentum defined by the unitarity conditions.

$$\mathrm{Res}_{ij\cdots k}\left[F(l)\right] \equiv \left[d_i(l)d_j(l)\cdots d_k(l)F(l)\right]\Big|_{l=l_{ij\cdots k}} \; .$$

$$\bar{d}_{ijkl}(l) = \mathrm{Res}_{ijkl}\left(\mathcal{A}_N(l)\right)$$ $d_i=d_j=d_k=d_l=0$ two solutions

$$\bar{c}_{ijk}(l) = \mathrm{Res}_{ijk}\left(\mathcal{A}_N(l) - \sum_{l\neq i,j,k}\frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)$$ $d_i=d_j=d_k=0$

infinite solutions

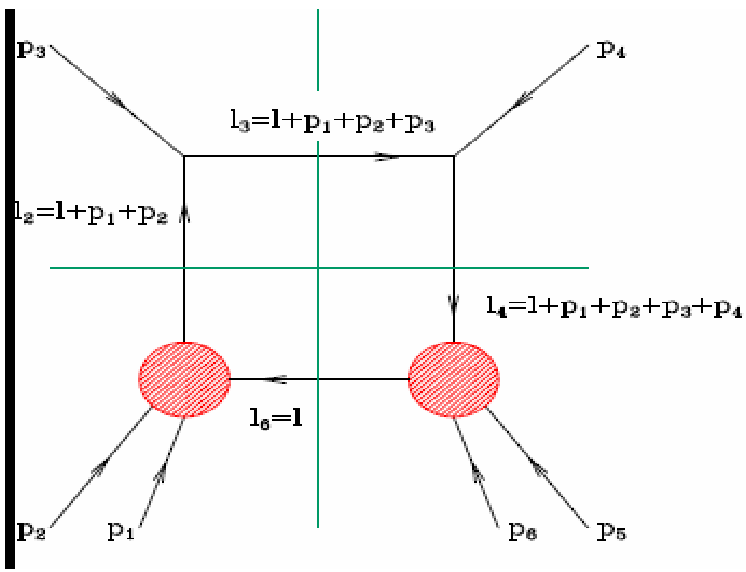
$$\bar{b}_{ij}(l) = \mathrm{Res}_{ij}\left(\mathcal{A}_N(l) - \sum_{k\neq i,j}\frac{\bar{c}_{ijk}(l)}{d_i d_j d_k} - \frac{1}{2!}\sum_{k,l\neq i,j}\frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)$$ $d_i=d_j=0$

The residue of the amplitudes factorize to the product of tree amplitudes

The box residue

$$\text{Res}_{2346}\Big(\mathcal{A}_6(l^{\pm})\Big) = \mathcal{M}_4^{(0)}(l_6^{\pm};p_1,p_2;-l_2^{\pm}) \times \mathcal{M}_3^{(0)}(l_2^{\pm};p_3;-l_3^{\pm})\mathcal{M}_3^{(0)}(l_3^{\pm};p_4;-l_4^{\pm})$$

$$\times\mathcal{M}_4^{(0)}(l_4^{\pm};p_5,p_6;-l_6^{\pm}) \quad = \quad \bar{d}_{ijkl}(l) = d_{ijkl} + \tilde{d}_{ijkl}\, l\cdot n_1$$



$$d_{ijkl} = \frac{\text{Res}_{ijkl}\Big(\mathcal{A}_N(l^+)\Big) + \text{Res}_{ijkl}\Big(\mathcal{A}_N(l^-)\Big)}{2}$$

$$\tilde{d}_{ijkl} = \frac{\text{Res}_{ijkl}\Big(\mathcal{A}_N(l^+)\Big) - \text{Res}_{ijkl}\Big(\mathcal{A}_N(l^-)\Big)}{2i\sqrt{V_4^2 - m_l^2}}$$

Left hand side: residua of the poles;

Right hand side: sum over factorized tree amplitudes

Numerical Implementation

Check the singular parts:

$$m^{(1)}(1,2,\ldots,n) \sim \left(-\frac{n}{\epsilon^2} + \frac{1}{\epsilon}\left(-\frac{11}{3} + \sum_{i=1}^{n} \log\left(\frac{s_{i,i+1}}{\mu^2}\right)\right)\right) \times m^{(0)}(1,2,\ldots,n) + \mathcal{O}(1)$$

Compare CPU time with those of the traditional method in case of 6g, 5g ,… amplitudes

EGZ: 9s per ordered amplitude on 2.8GHz Pentium processor
EGK: 0.01s per ordered amplitude on 2.8GHz Pentium processor

	ev.time	# of cuts
4 gluon:	0.0009s	6
5 gluon:	0.0035s	20
6 gluon :	0.0107s	44

Computer time: scales with $\approx$ n^4 (# of cuts) not as n!

Numerical Unitarity Method in D-dimension for gluon amplitudes

Two sources of D-dependence

i) spin-polarization states live in D_s .

ii) loop momentum component live in D. (D_s>D)

$$\mathcal{A}_{(D,D_s)}(\{p_i\},\{J_i\}) = \int \frac{d^D l}{i(\pi)^{D/2}} \frac{\mathcal{N}^{(D_s)}(\{p_i\},\{J_i\};l)}{d_1 d_2 \cdots d_N}.$$

$$\sum_{i=1}^{D_s-2} e_\mu^{(i)}(l) e_\nu^{(i)}(l) = -g_{\mu\nu}^{(D_s)} + \frac{l_\mu b_\nu + b_\mu l_\nu}{l \cdot b},$$

$$l^2 = \bar{l}^2 - \tilde{l}^2 = l_1^2 - l_2^2 - l_3^2 - l_4^2 - \sum_{i=5}^{D} l_i^2$$

Two key features

Dependence on Ds is linear

$$\mathcal{N}^{(D_s)}(l) = \mathcal{N}_0(l) + (D_s - 4)\mathcal{N}_1(l)$$

- Choose two integer values $D_s = D_1$ and $D_s = D_2$ to reconstruct the full D_s dependence.
- Suitable for numerical implementation.
- $D_s=4-2\varepsilon$ ‘t Hooft Veltman scheme, $D_s=4$ FDHS

The loop momentum effectively has only 4+1 component

$$N(l) = N(l_4, \mu^2) \qquad \mu^2 = -l_5^2 - \ldots - l_D^2$$

maximum 5 constraints: we need to consider also pentagon cuts.

Reduction in D-dimensions

The parametrization of the N-particle amplitude

$$\frac{\mathcal{N}^{(D_s)}(l)}{d_1 d_2 \cdots d_N} = \sum_{[i_1|i_5]} \frac{\overline{e}^{(D_s)}_{i_1 i_2 i_3 i_4 i_5}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} + \sum_{[i_1|i_4]} \frac{\overline{d}^{(D_s)}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}}$$

$$+ \sum_{[i_1|i_3]} \frac{\overline{c}^{(D_s)}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_2]} \frac{\overline{b}^{(D_s)}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{[i_1|i_1]} \frac{\overline{a}^{(D_s)}_{i_1}(l)}{d_{i_1}}$$

Parametrization of the residues

Pentuple residue: $\overline{e}^{(D_s)}_{ijkmn}(l) = e^{(D_s,(0))}_{ijkmn}$

Box residue: $\overline{d}^{\mathrm{FDH}}_{ijkn}(l) = d^{(0)}_{ijkn} + d^{(1)}_{ijkn} s_1 + (d^{(2)}_{ijkn} + d^{(3)}_{ijkn} s_1) s_e^2 + d^{(4)}_{ijkn} s_e^4$

Three extra structures for triple, three for double and zero for single cuts, only even powers of $s_e = n_e\, l$

Four new master integrals

Four of the s_e^2 dependent master integrals are not spurious

$$\int \frac{d^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} = -\frac{D-4}{2} I^{D+2}_{i_1 i_2 i_3 i_4}, \quad \ldots$$

We obtain the full D-dependence of the amplitude

$$\mathcal{A}_{(D)} = \sum_{[i_1|i_5]} e^{(0)}_{i_1 i_2 i_3 i_4 i_5} \; I^{(D)}_{i_1 i_2 i_3 i_4 i_5}$$

$$+ \sum_{[i_1|i_4]} \left(d^{(0)}_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} - \frac{D-4}{2} d^{(2)}_{i_1 i_2 i_3 i_4} \; I^{(D+2)}_{i_1 i_2 i_3 i_4} + \frac{(D-4)(D-2)}{4} d^{(4)}_{i_1 i_2 i_3 i_4} \; I^{(D+4)}_{i_1 i_2 i_3 i_4} \right)$$

+ similar terms for triangle, bubble and tadpole contributions.

As ε -> 0 the new master integrals can be decomposed in the old basis and generate ε dependent bubble coefficients !

One-loop amplitudes up to terms of order ε

One loop amplitudes as sum of cut-constructible and rational parts:

$$\mathcal{A}_N = \mathcal{A}_N^{CC} + R_N.$$

The cut constructible part is as before (EGK):

$$\mathcal{A}_N^{CC} = \sum_{[i_1|i_4]} \tilde{d}^{(0)}_{i_1 i_2 i_3 i_4} \; I^{(4-2\epsilon)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c^{(0)}_{i_1 i_2 i_3} \; I^{(4-2\epsilon)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b^{(0)}_{i_1 i_2} \; I^{(4-2\epsilon)}_{i_1 i_2} + \sum_{i_1=1}^{N} a^{(0)}_{i_1} \; I^{(4-2\epsilon)}_{i_1},$$

The rational part is new (GKM):

$$R_N = -\sum_{[i_1|i_4]} \frac{d^{(4)}_{i_1 i_2 i_3 i_4}}{3} - \sum_{[i_1|i_3]} \frac{c^{(9)}_{i_1 i_2 i_3}}{2} - \sum_{[i_1|i_2]} \left(\frac{(q_{i_1} - q_{i_2})^2}{6} - \frac{m_{i_1}^2 + m_{i_2}^2}{2} \right) b^{(9)}_{i_1 i_2}.$$

Numerical evaluation of 4g,5g,6g amplitudes

- **Choose D_1=5 and D_2=6:** $$\mathcal{A}^{\mathrm{FDH}} = 2\mathcal{A}_{(D,D_s=5)} - \mathcal{A}_{(D,D_s=6)}$$

- **no use supersymmetry;**
- **tree amplitudes are calculated with Berends-Giele recursion relations in D_s=5 and D_s=6 dimensions;**
- **numerical evaluation in maple (GKM) and in ROCKET (Giele Zanderighi)**

Comparison with

i) known analytic results (Bern,Kosower, Britto; Feng, Mastrolia)
ii) known semi-numerical results (IBP) (Ellis, Giele, Zanderighi)

Numerical Evaluation using Maple

Computer time: scales with # of cuts (n^4) and not as n!

Calculation of the rational part increases the computer time with a factor less than 2

CPU is independent from helicity configurations

$\lambda_1, \lambda_2, \ldots, \lambda_6$	Δ^{cut}	Δ^{rat}	Δ
- - + + + +	-19.481065+78.147162 i	28.508591-74.507275 i	9.027526+3.639887 i
- + - + + +	-241.10930+27.176200 i	250.27357-25.695269 i	9.164272+1.480930 i
- + + - + +	5.4801516-12.433657 i	0.19703574+0.25452928 i	5.677187-12.179127 i
- - - + + +	15.478408-2.7380153 i	2.2486654+1.0766607 i	17.727073-1.661354 i
- - + - + +	-339.15056-328.58047 i	348.65907+336.44983 i	9.508509+7.869351 i
- + - + - +	31.947346+507.44665 i	-17.430910-510.42171 i	14.516436-2.975062 i

TABLE III: Finite parts of singular six-gluon scattering amplitudes for various gluon helicities.

Numerical Evaluation using Rocket

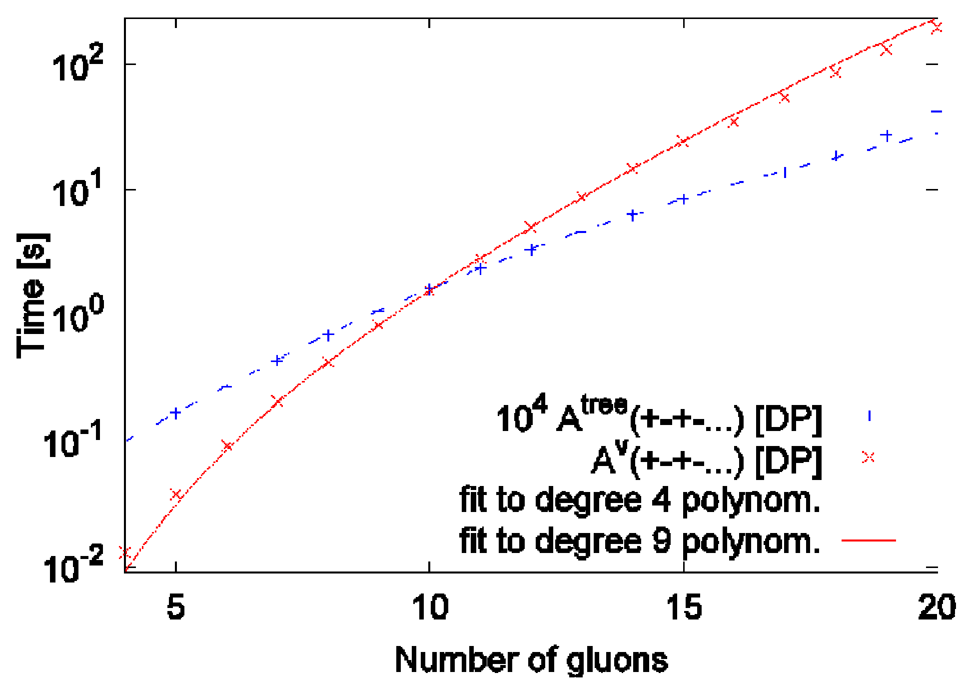
Fortran code by Giele Zanderighi

The predicted scaling of the polynomial growth of the computer time tested for color ordered subamplitudes up to 20 legs.

Time for large N: $\tau(\text{tree}) \approx N^4$ $\tau(\text{loop}) \approx N^9$

factor ≈ 30 increase from doubleP to QP

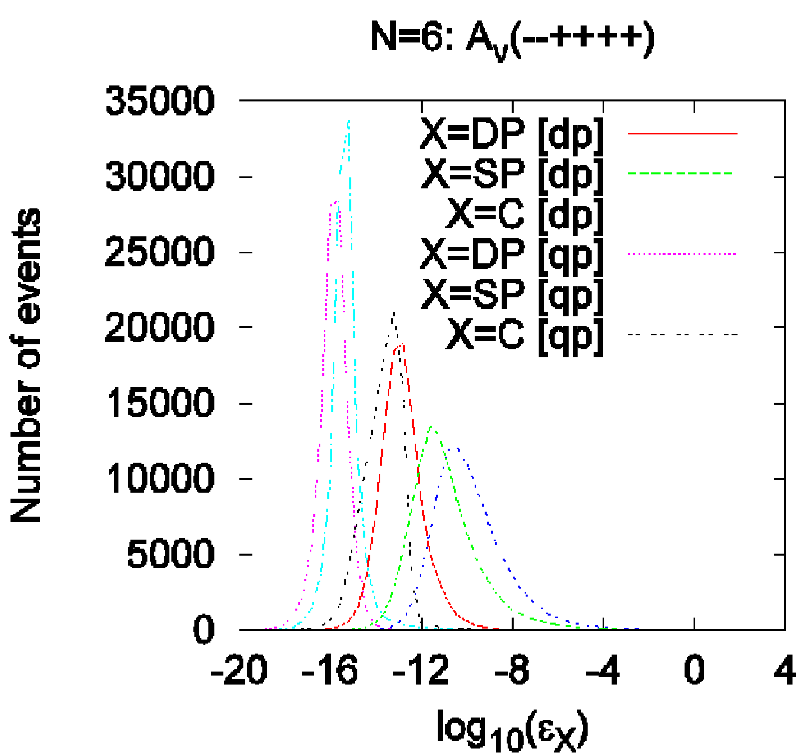
2.33GHz Xeon proc
6gluon ≈ 90ms

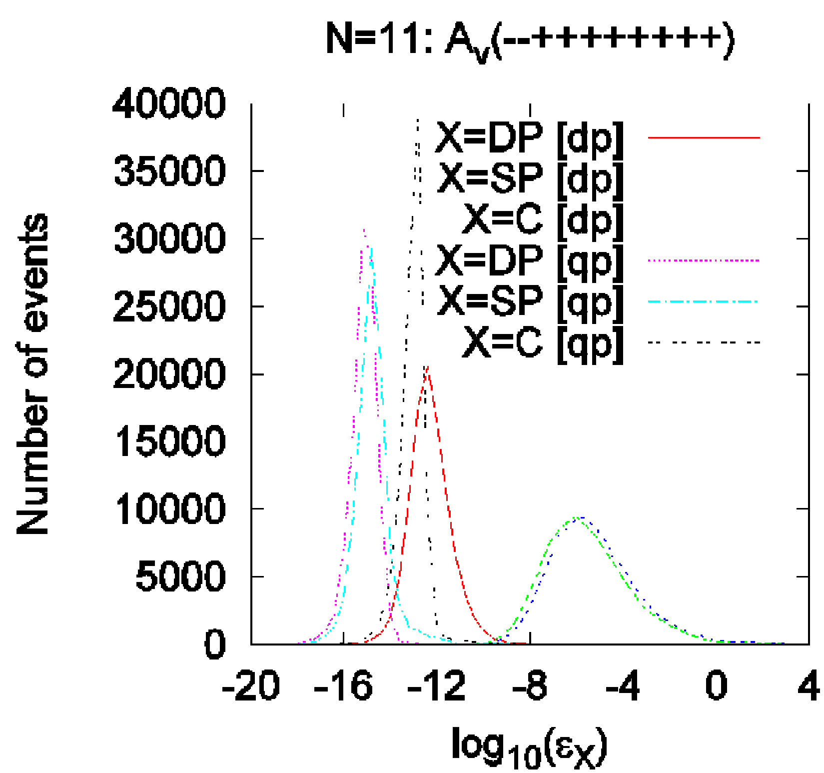


Numerical Evaluation using Black Hat

Talk by D. Kosower

Numerical results   MHV amplitudes





$$S = \log_{10}\left(\left|\frac{m^{(1)}_{\text{unitarity}} - m^{(1)}_{\text{analytic}}}{m^{(1)}_{\text{analytic}}}\right|\right) .$$

Numerical D-dimensional unitary algorithm for massive fermions

Application to ggtt and gggtt

- **We have to choose even values for D_s**

$$\mathcal{A}^{\mathrm{FDH}} = 2\mathcal{A}_{(D,D_s=6)} - \mathcal{A}_{(D,D_s=8)}$$

- **Pentagon, box, triangle ,bubble and tadpole cuts**
- **The treatment of bubble and tadpole cuts is more subtle: difficulty with self-energy insertions on external lines.**
- **Particles of different flavors: more sophisticated bookkeeping**
- **More master integrals (use Ellis Zanderighi)**

Gluon polarizations in 6 dimensions

$$\rho^{\mu\nu}(l,\eta) = -g^{\mu\nu} + \frac{(l^\mu \eta^\nu + l^\nu \eta^\nu)}{l\cdot\eta}, \qquad \omega^{\mu\nu} = -g^{\mu\nu} + \bar{l}^\mu \bar{l}^\nu / \bar{l}^2$$

If l is 4-dimensional

$$\sum_{i=1}^{4} e_i^\mu e_i^\nu = \begin{cases} \rho^{\mu\nu}(l,\eta), & \mu,\nu \in 4\ \text{dim}; \\ -n_5^\mu n_5^\nu, & \mu=5,\nu=5; \\ -n_6^\mu n_6^\nu, & \mu=6,\nu=6; \\ 0, & \text{otherwise.} \end{cases}$$

If l is 5-dimensional

$$\sum_{i=1}^{4} e_i^\mu e_i^\nu = \begin{cases} -\omega^{\mu\nu}(\bar{l}); & \mu,\nu \in 4\text{dim}; \\ -n_6^\mu n_6^\nu, & \mu=6,\nu=6, \\ 0, & \text{otherwise.} \end{cases}$$

$$l^\mu = \bar{l}^\mu + \beta n_5^\mu$$

Massive Dirac spinors in 6 dimensions

$$u^{(s)}(l,m) = \frac{(l_\mu \Gamma^\mu + m)}{\sqrt{l_0 + m}} \eta_{D_s}^{(s)}, \qquad s = 1, \ldots, 2^{D_s/2-1}$$

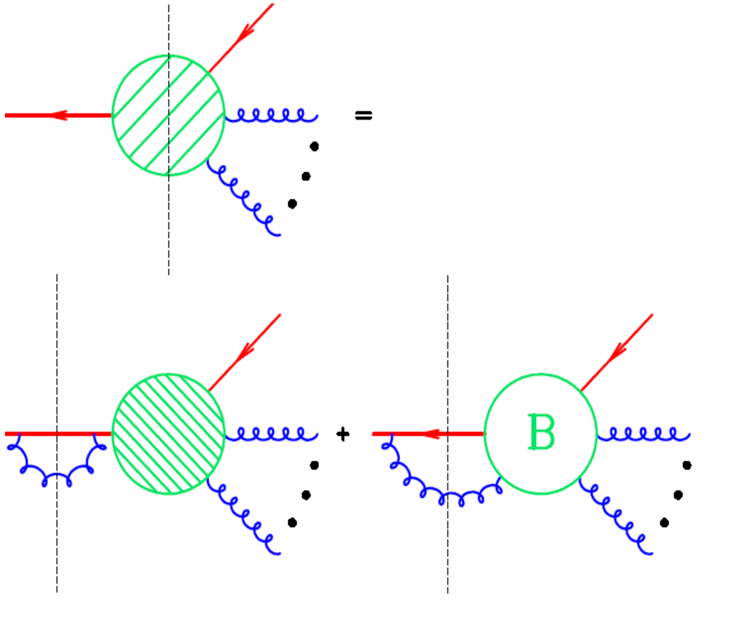
$$\eta_4^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \eta_4^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix},$$

$$\eta_6^{(1)} = \begin{pmatrix} \eta_4^{(1)} \\ 0 \end{pmatrix}, \quad \eta_6^{(2)} = \begin{pmatrix} \eta_4^{(2)} \\ 0 \end{pmatrix}, \quad \eta_6^{(3)} = \begin{pmatrix} 0 \\ \eta_4^{(1)} \end{pmatrix}, \quad \eta_6^{(4)} = \begin{pmatrix} 0 \\ \eta_4^{(2)} \end{pmatrix}.$$

$$\bar{u}^{(s)}(l,m) = \bar{\eta}_{D_s}^{(s)} \frac{(l_\mu \Gamma^\mu + m)}{\sqrt{l_0 + m}}$$

l_μ is not conjugated

Self-energy on external massive fermion leg



For massless line: vanishing contributions

Double count of the external self-energy
Tree amplitude on the right hand side is not well defined

$$\mathrm{Res}\left[A^{[1]}(t,g_1,\ldots,g_n,\bar{t})\right] \sim \sum_{\mathrm{states}} A^{[0]}(t,g^*,\bar{t}^*) \times A^{[0]}(t^*,g^*,g_1,\ldots,g_n,\bar{t})$$

$$A^{[0]}(t^*,g^*,g_1,\ldots,g_n,\bar{t}) = \frac{R(t^*,g^*,g_1,\ldots,g_n,\bar{t})}{(p_{t*}+p_{g^*})^2 - m_t^2} + B(t^*,g^*,g_1,\ldots,g_n,\bar{t}).$$

Self-energy contribution, gauge invariance and generic conflict with unitarity

Feynman diagram calculation:

i) one particle reducible self-energy corrections on external legs are discarded

ii) Their effects are included by wave-function renormalization constants (Z_2)

Follow the same path:

i) discard the term in the tree amplitude generating one particle reducible diagrams

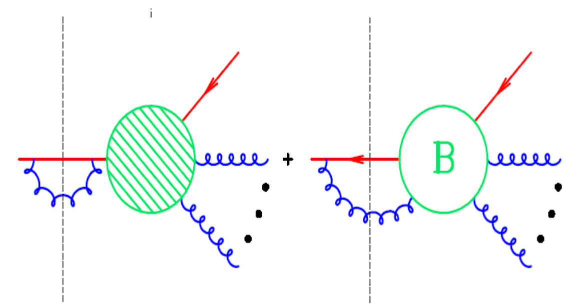
BG recursion relations can accommodate it by truncating the recursive steps

ii) It is taken into account by adding later wave function renormalization

The remaining part of the amplitude (B) is not gauge invariant

iii) **The gauges used to calculate Z_2 and B must be the same**

$$\sum_{s=1}^{D_s} e_s^\mu e_s^\nu = -g^{\mu\nu} \ .$$

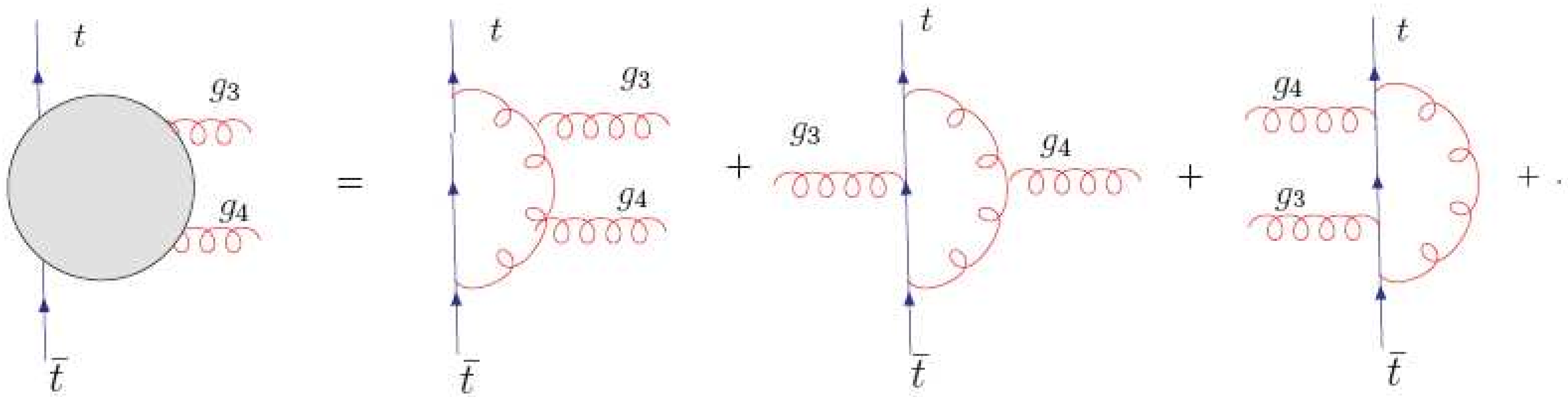


It mildly violates "unitarity ": sum over non-physical states

Primitive amplitudes

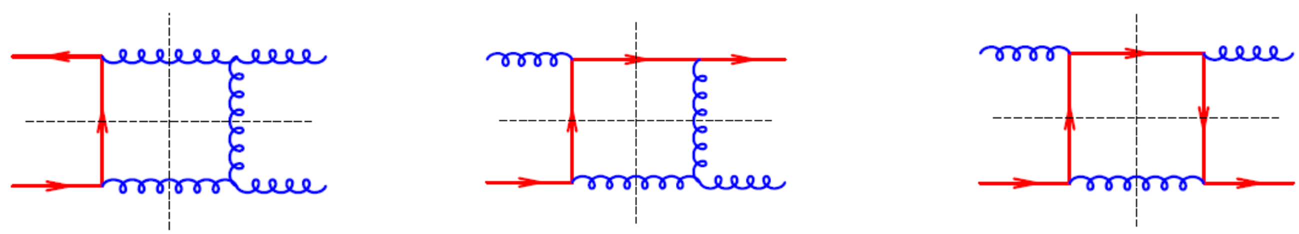
Bern, Dixon, Kosower (1994)

Special role: the flavor of the cut lines are uniquely defined



$A_L(1_{\bar{t}}, 2_t, 3, 4)$ $A_L(1_{\bar{t}}, 3, 2_t, 4)$ $A_L(1_{\bar{t}}, 3, 4, 2_t)$

Three distinct quadruple cuts → three gauge invariant primitive amplitudes



Numerical evaluation of the primitive amplitudes for ttgg and ttggg

$$A_L(1_{\bar{t}}, 2_t, g_1, g_2) \quad A_L(1_{\bar{t}}, g_1, 2_t, g_2) \quad A_L(1_{\bar{t}}, g_1, g_2, 2_t)$$

$$A_L(1_{\bar{t}}, 2_t, g_1, g_2, g_3) \quad A_L(1_{\bar{t}}, g_1, 2_t, g_2, g_3) \quad A_L(1_{\bar{t}}, g_1, g_2, 2_t, g_3) \quad A_L(1_{\bar{t}}, g_1, g_2, g_3, 2_t)$$

INPUT

i) Born primitive amplitudes are calculated using BG recursion

tadpole cuts: $\bar{t}t + \bar{t}t + 2, 3$ six-, seven-leg tree amplitudes

ii) we have to calculate renormalized one loop primitive amplitudes

Z_2, Z_m factors + mass counter term diagrams (restores gauge invariance)

iii) test: correct soft collinear limits, + traditional calculation

iv) Master integral input from Ellis Zanderighi

Results for the $\bar{t}t + 3$ gluons tree and loop primitive amplitudes

Computer time from ggtt to gggtt scales the same way as in case of gluons

Fortran77 code

Amplitude	tree	c^{cut}	c
$+_{\bar{t}}, +_t, +_3, +_4, +_5$	-0.000533-0.000137 i	9.584144+6.530925 i	51.8809+6.543042 i
$+_{\bar{t}}, -_t, +_3, -_4, +_5$	-0.004540 + 0.018665 i	19.65913-11.77003 i	23.00306-9.699584 i
$+_{\bar{t}}, +_t, -_3, +_4, -_5$	-0.004726+ 0.014201 i	33.15950-1.832717 i	33.71943 -3.142751 i
$+_{\bar{t}}, -_t, -_3, +_4, +_5$	0.045786 + 0.010661 i	22.84043-6.540697 i	23.03114-7.313041 i
$+_{\bar{t}}, +_3, +_t, +_4, +_5$	0.000182 + 0.001369 i	6.517366-1.277070 i	19.37656+7.563101 i
$+_{\bar{t}}, +_3, -_t, -_4, +_5$	0.0467366-0.006020 i	19.440997-7.639466 i	20.93024-9.936409 i
$+_{\bar{t}}, -_3, +_t, +_4, -_5$	0.019275 -0.0732138 i	15.31910 -3.9278496 i	15.176306-4.102803 i
$+_{\bar{t}}, -_3, -_t, +_4, +_5$	-0.018203-0.111312 i	24.13158+1.431596 i	24.70002+1.018096 i
$+_{\bar{t}}, +_3, +_4, +_t, +_5$	0.00060-0.001377 i	13.13854+6.157043 i	10.13113+13.83997 i
$+_{\bar{t}}, +_3, -_4, -_t, +_5$	-0.047199-0.021516 i	23.90539 -2.168867 i	22.905695-4.284617 i
$+_{\bar{t}}, -_3, +_4, +_t, -_5$	-0.015110+0.063118 i	13.54258-7.800591 i	13.50273-8.018127 i
$+_{\bar{t}}, -_3, +_4, -_t, +_5$	-0.048800+ 0.112645 i	21.77602+ 2.078051 i	22.52784+1.424481 i
$+_{\bar{t}}, +_3, +_4, +_5, +_t$	-0.000252+0.000144 i	-10.35085+45.26276 i	-98.81384+52.81712 i
$+_{\bar{t}}, +_3, -_4, +_5, -_t$	0.0050023+0.008871 i	23.944473+2.862220 i	20.92683-0.968026 i
$+_{\bar{t}}, -_3, +_4, -_5, +_t$	0.000561-0.004105 i	-2.987822-42.00048 i	-3.834451-43.67103 i
$+_{\bar{t}}, -_3, +_4, +_5, -_t$	0.021216-0.011994 i	19.72995-2.120128 i	20.94996-1.684734 i

Concluding remarks

- Numerical D-dimensional Unitarity approach is a robust algorithm to evaluate efficiently one loop amplitudes in QCD

- It will allow us to evaluate NLO corrections for a large number of high multiplicity processes with particles of various mass, spin and flavor values relevant for LHC phenomenology.