# MSYM amplitudes in the high-energy limit

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INFN LNF

Gauge theory and String theory

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# In principio erat Bern-Dixon-Smirnov ansatz ...

an ansatz for MHV amplitudes in N=4 SUSY

Bern Dixon Smirnov 05

$$m_n = m_n^{(0)} \left[ 1 + \sum_{L=1}^{\infty} a^L M_n^{(L)}(\epsilon) \right]$$

$$= m_n^{(0)} \exp \left[ \sum_{l=1}^{\infty} a^l \left( f^{(l)}(\epsilon) M_n^{(1)}(l\epsilon) + Const^{(l)} + E_n^{(l)}(\epsilon) \right) \right]$$

coupling $a = \frac{\lambda}{8\pi^2}(4\pi e^{-\gamma})^\epsilon$ $\lambda = g_s^2 N_c$ 't Hooft parameter

$$f^{(l)}(\epsilon) = \frac{\hat{\gamma}_K^{(l)}}{4} + \frac{l}{2}\hat{G}^{(l)}\epsilon + f_2^{(l)}\epsilon^2 \qquad E_n^{(l)}(\epsilon) = O(\epsilon)$$

$\hat{\gamma}_K^{(l)}$ cusp anomalous dimension, known to all orders of *a*

Korchemsky Radyuskin 86
Beisert Eden Staudacher 06

$\hat{G}^{(l)}$ IR function, known through O($a^4$)

Bern Dixon Smirnov 05
Cachazo Spradlin Volovich 07

# Brief history of BDS ansatz

BDS ansatz checked through 3-loop 4-pt amplitude — Bern Dixon Smirnov 05

2-loop 5-pt amplitude — Cachazo Spradlin Volovich 06; Bern Czakon Kosower Roiban Smirnov 06

BDS ansatz shown to fail on 2-loop 6-pt amplitude

Bern Dixon Kosower Roiban Spradlin Vergu Volovich 08

Hints of break-up also from strong-coupling expansion — Alday Maldacena 07

hexagon Wilson loop — Drummond Henn Korchemsky Sokatchev 07

multi-Regge limit — Bartels Lipatov Sabio-Vera 08

# BDS ansatz and Regge limit

4-pt amplitude $p_a p_b \to p_{a'} p_{b'}$ in the Regge limit $s \gg -t$

$$m_4 = s\,[g_s\,C(p_a,p_{a'})]\,\frac{1}{t}\left(\frac{s}{-t}\right)^{\alpha(t)}[g_s\,C(p_b,p_{b'})]$$

$\alpha(t)$ Regge trajectory $C(p_a,p_{a'})$ coefficient function

$$\alpha(t,\epsilon) = \sum_{l=1}^{\infty} \bar{g}_s^{2l}(t,\epsilon)\alpha^{(l)}(\epsilon)$$

$$\bar{g}_s^2(t,\epsilon) = \frac{a}{2G(\epsilon)}\left(\frac{\mu^2}{-t}\right)^{\epsilon} \qquad G(\epsilon) = \frac{e^{-\gamma\epsilon}\,\Gamma(1-2\epsilon)}{\Gamma(1+\epsilon)\,\Gamma^2(1-\epsilon)} = 1 + O(\epsilon^2)$$

Because the Regge limit is exponential in the Regge trajectory, one can use (the logarithm of) the BDS ansatz to obtain the Regge trajectory to all loops

Naculich Schnitzer 07
Bartels Lipatov Sabio-Vera 08
Glover VDD 08

*l*-loop Regge trajectory

$$\alpha^{(l)}(\epsilon) = 2^{l-1}\alpha^{(1)}(l\epsilon)\left(\frac{\hat{\gamma}_K^{(l)}}{4} + \frac{l}{2}\hat{G}^{(l)}\epsilon\right) + O(\epsilon) \qquad \alpha^{(1)}(\epsilon) = \frac{2}{\epsilon}$$

the BDS ansatz can also be used to compute
(or to derive relations between) the coefficient functions

High-energy factorisation is valid also for amplitudes with 5 or more points
in generalised Regge limits.
The general strategy is to use the modular form
of the amplitudes dictated by high-energy factorisation,
to obtain information on *n*-point amplitudes in terms of building blocks derived
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Because high-energy factorisation is used in the derivation
in QCD of the BFKL equation at LL and NLL accuracy,
I will start from there
with a few slides of a few years ago ...

# Forward Scattering

**Parton-parton scattering** In the c.m. frame, $t = -s(1 - \cos\theta)/2$, with $\theta$ the scattering angle. $s \gg |t|$:

- forward, i.e. small angle, scattering: $d\sigma/dt \sim 1/t^2$
- the scattering process is dominated by sub-processes with gluon exchange in the $t$ channel: $q\ Q \rightarrow q\ Q$, $q\ g \rightarrow q\ g$, $g\ g \rightarrow g\ g$

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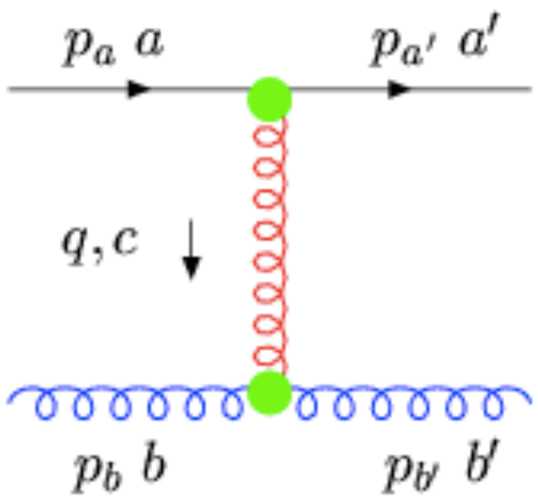
$q\,g \to q\,g$ scattering amplitude in the $s \gg |t|$ limit:




$$\mathcal{A}^{\text{tree}}_{q\,g\to q\,g}(p_a, p_{a'}|p_{b'}, p_b) = 2s\left[g\,T^c_{a'\bar{a}}\,C^{q;q}(p_a;p_{a'})\right]\frac{1}{t}\left[ig\,f^{bb'c}\,C^{g;g}(p_b;p_{b'})\right]$$

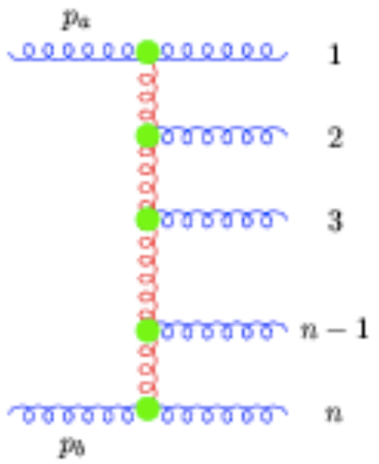
- $C^{g;g}$ ($C^{q;q}$): gluon (quark) high energy effective vertices
- high energy factorisation: to obtain $q\,Q \to q\,Q$ or $g\,g \to g\,g$ replace

$$ig\,f^{bb'c}\,C^{g;g}(p_b;p_{b'}) \leftrightarrow g\,T^c_{b'\bar{b}}\,C^{q;q}(p_b;p_{b'})$$

# BFKL Resummation

- in any scattering process with $s \gg |t|$ gluon exchange in the $t$ channel dominates

- BFKL is a resummation of multiple gluon radiation out of the gluon exchanged in the $t$ channel




- for $s \gg |t|$ BFKL resums the Leading Log (and Next-to-Leading Log) contributions, in $\log(s/t)$, of the radiative corrections to the gluon propagator in the $t$ channel, to all orders in $\alpha_s$

- the LL terms are obtained in the approximation of strong rapidity ordering $(y_1 \gg y_2 \gg \ldots \gg y_n)$ and no $k_t$ ordering of the emitted gluons

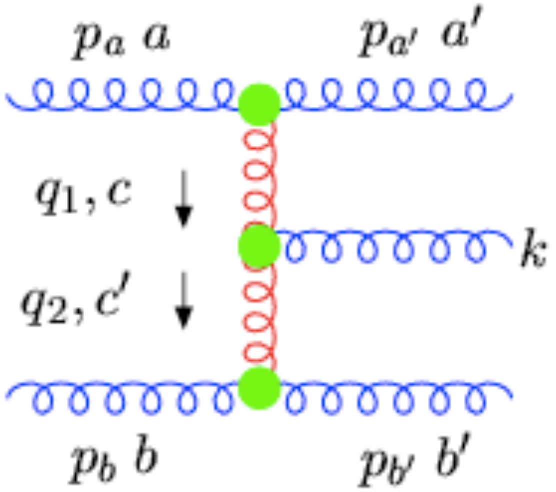
- the NLL terms are universal

- the resummation yields a 2-dim integral equation for the evolution of the gluon propagator in the $t$ channel

# LL BFKL Resummation

* the universal building blocks of the LL BFKL resummation are:

  * the real term: the emission of a gluon along the gluon ladder




$$\mathcal{A}^{\rm tree}_{gg\to 3g}(p_a, p_{a'}|k|p_{b'}, p_b) = s\left[ig\, f^{aa'c}\, C^{g;g}(p_a; p_{a'})\right] \times \frac{1}{t_1}\left[ig\, f^{cdc'}\, C^{g}(q_1, k, q_2)\right] \times \frac{1}{t_2}\left[ig\, f^{bb'c'}\, C^{g;g}(p_b; p_{b'})\right]$$

➡ $C^g(q_1, k, q_2)$ is the gluon emission (Lipatov) vertex

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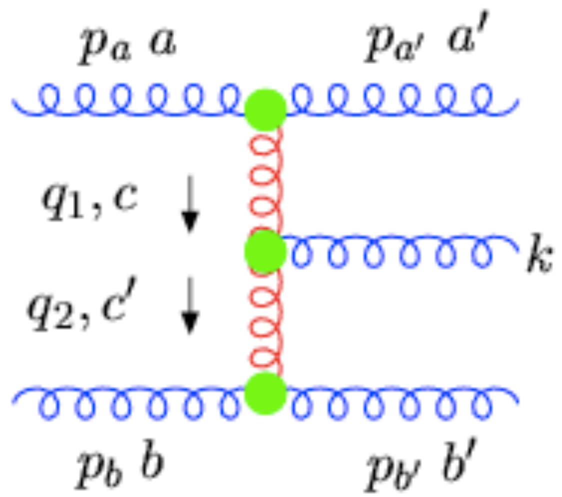


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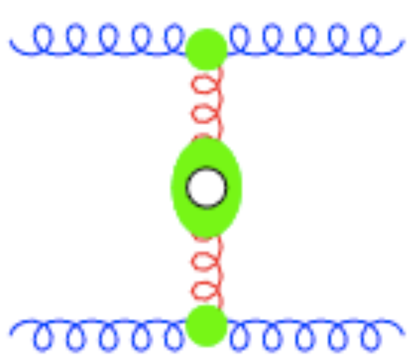
➡ $C^g(q_1, k, q_2)$ is the gluon emission (Lipatov) vertex

- the virtual term: the reggeisation of the gluon exchanged in the $t$ channel (here in $d = 4 - 2\epsilon$ dimensional regularisation)



$$\mathcal{A}^{1-\text{loop}}_{g\,g\to g\,g} = \tilde{g}^2(t)\alpha^{(1)} \ln\frac{s}{-t}\mathcal{A}^{\text{tree}}_{g\,g\to g\,g}$$

$$\alpha^{(1)} = \frac{2C_A}{\epsilon} \qquad \tilde{g}^2(t) = g^2 c_\Gamma \left(\frac{\mu^2}{-t}\right)^{\epsilon}$$

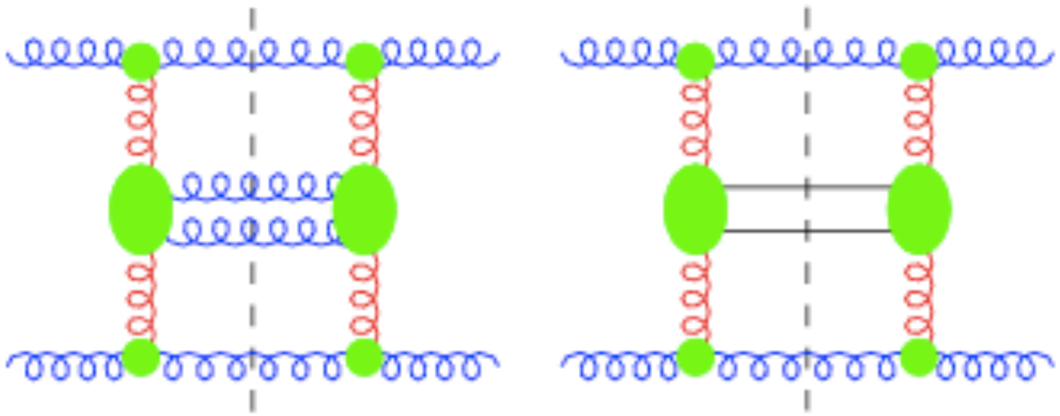
➡ $\tilde{g}^2(t)\alpha^{(1)}$ is the 1-loop gluon Regge trajectory $(C_A = N_c)$

# NLL BFKL RESUMMATION

* the building blocks of the NLL BFKL resummation are:

- corrections to the Lipatov vertex

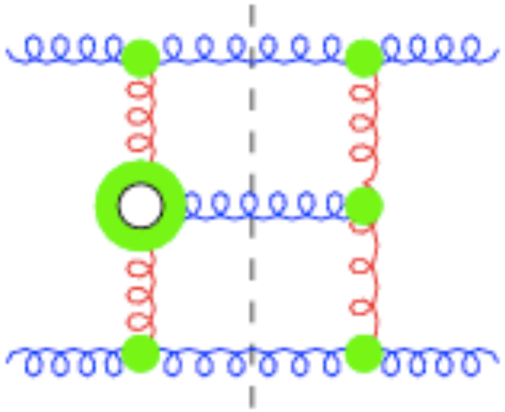
real



Fadin, Lipatov 1989-96

VDD 1996

virtual



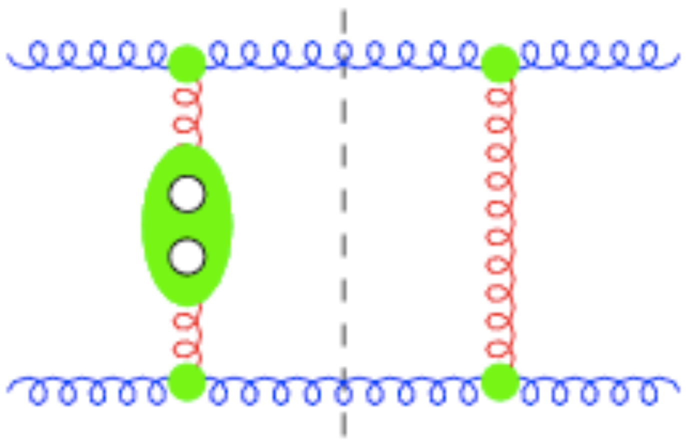
Fadin, Lipatov 1993

Fadin, Fiore, Quartarolo 1994

Fadin, Fiore, Kotsky 1996

Bern, Schmidt, VDD 1998

- 2-loop gluon reggeisation



Fadin, Fiore, Kotsky 1995-96

Fadin, Fiore, Quartarolo 1995

Glover, VDD 2001

# GLUON REGGEISATION

**ANSATZ** in HEL the gluon-gluon scattering amplitude for the exchange of a colour octet of negative signature in the $t$ channel is

$$\mathcal{A}_{g\,g\to g\,g}(p_a, p_{a'}|p_{b'}, p_b)$$
$$= \quad s\left[ig\, f^{aa'c}\, C^{g;g}(p_a;p_{a'})\right]\frac{1}{t}\left[\left(\frac{-s}{-t}\right)^{\alpha(t)}+\left(\frac{s}{-t}\right)^{\alpha(t)}\right]\left[ig\, f^{bb'c}\, C^{g;g}(p_b;p_{b'})\right]$$

- the effective vertex $C^{g;g}$ and the gluon Regge trajectory have the perturbative expansion

$$C^{g;g} = C^{g;g(0)}(1+\tilde{g}^2(t)C^{g;g(1)}+\tilde{g}^4(t)C^{g;g(2)})+\mathcal{O}(\tilde{g}^6)$$
$$\alpha(t) = \tilde{g}^2(t)\alpha^{(1)}+\tilde{g}^4(t)\alpha^{(2)}+\mathcal{O}(\tilde{g}^6)$$

- the 2-loop gluon Regge trajectory is

$$\alpha^{(2)} = C_A\left[\beta_0\frac{1}{\epsilon^2}+K\frac{2}{\epsilon}+C_A\left(\frac{404}{27}-2\zeta_3\right)+N_F\left(-\frac{56}{27}\right)\right]$$

where $\beta_0 = \frac{(11C_A-2N_F)}{3}$ $\quad K = \left(\frac{67}{18}-\frac{\pi^2}{6}\right)C_A - \frac{5}{9}N_F$

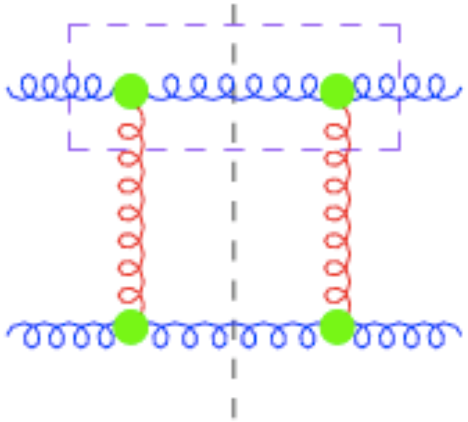
maximal trascendentality

Kotikov Lipatov 02

# IMPACT FACTORS

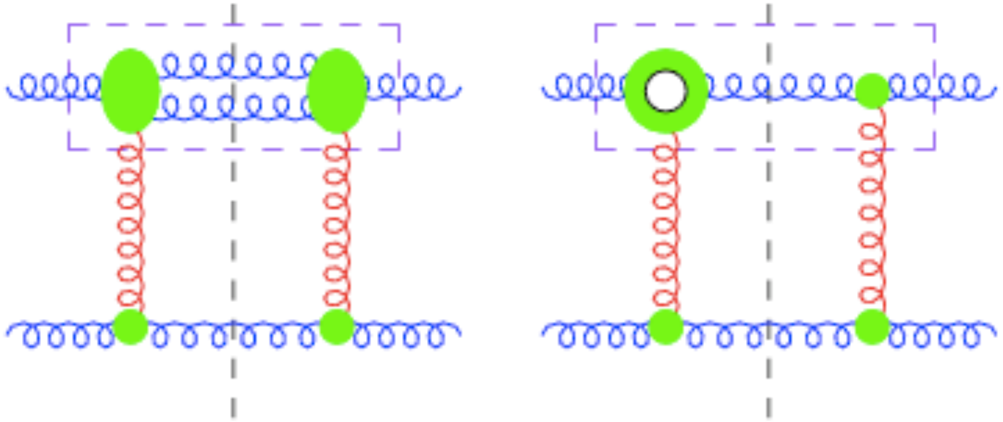
**LO IMPACT FACTOR**

$g\, g^* \to g$:



☛ at LO the impact factors are known for all the processes of interest (see next Table)

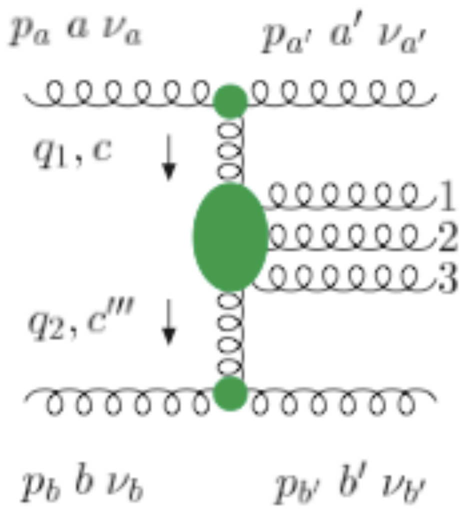
**NLO IMPACT FACTOR**



☛ at NLO the impact factors are known for $q\, g^* \to q$, $g\, g^* \to g$ and $\gamma^* g^* \to q\bar{q}$

Bartels, Colferai, Gieseke, Vacca 2001-02

# More tree coefficient functions ...




contributes to NNLL BFKL kernel

Frizzo Maltoni VDD 99

Antonov Lipatov Kuraev Cherednikov 05

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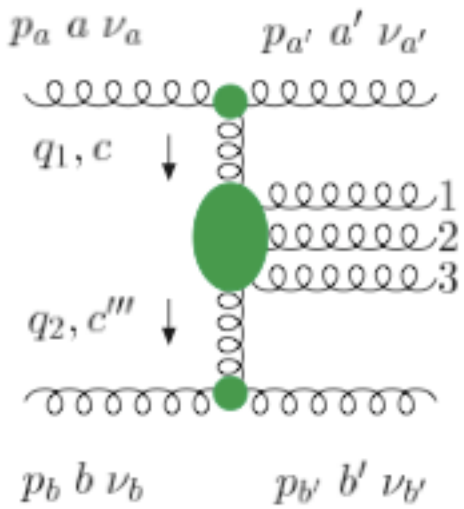


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Frizzo Maltoni VDD 99

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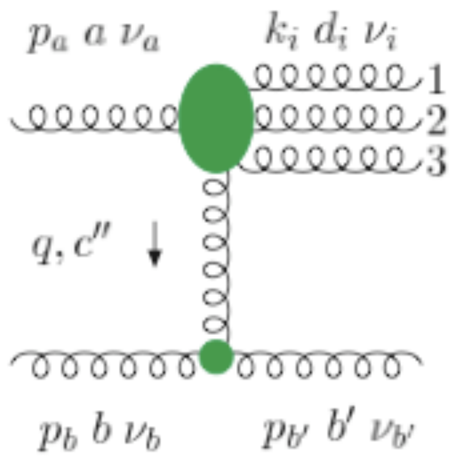


contributes to NNLO impact factor
(boundary condition to NNLL kernel)

Frizzo Maltoni VDD 99

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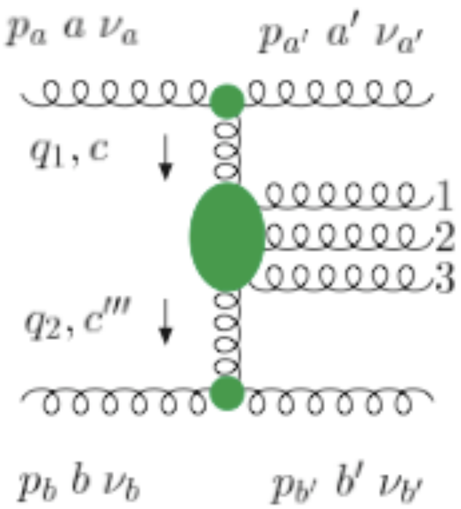


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Frizzo Maltoni VDD 99

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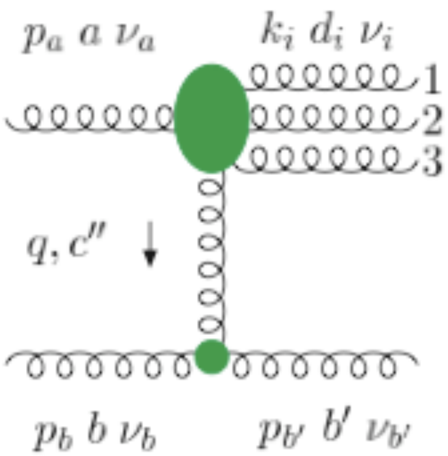
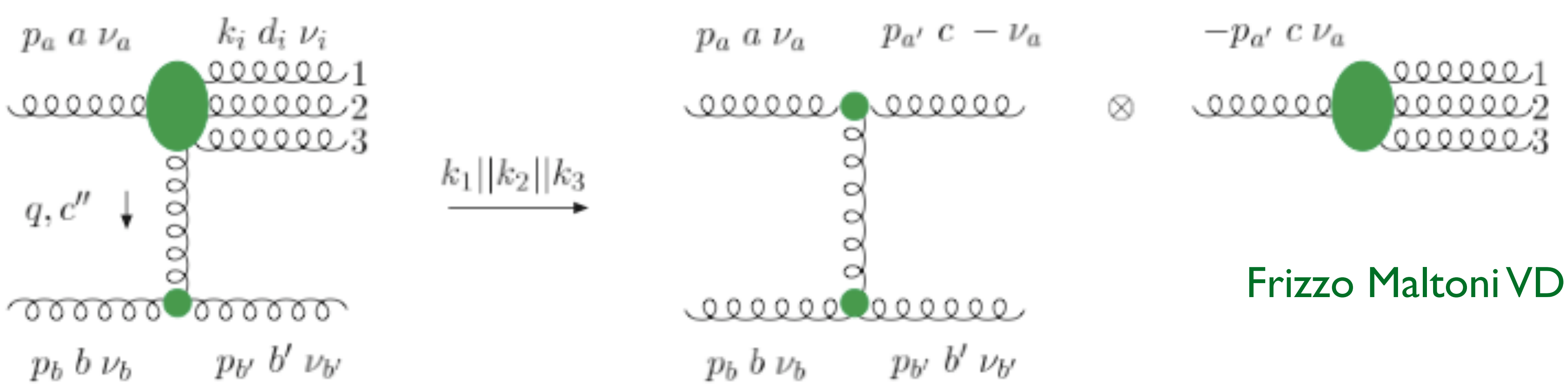



contributes to NNLO impact factor
(boundary condition to NNLL kernel)

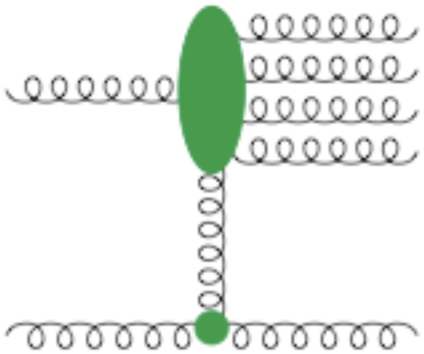
Frizzo Maltoni VDD 99

used to compute DGLAP splitting amplitudes for all parton species




Frizzo Maltoni VDD 99

# Tree 4-gluon coefficient function



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Frizzo Maltoni VDD 99

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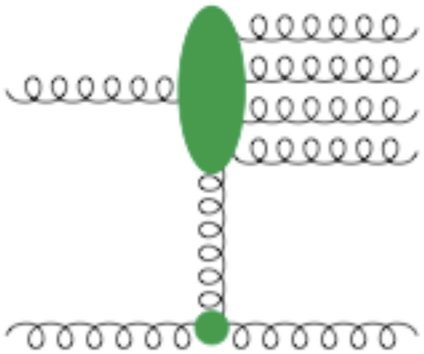

contributes to NNNLO impact factor
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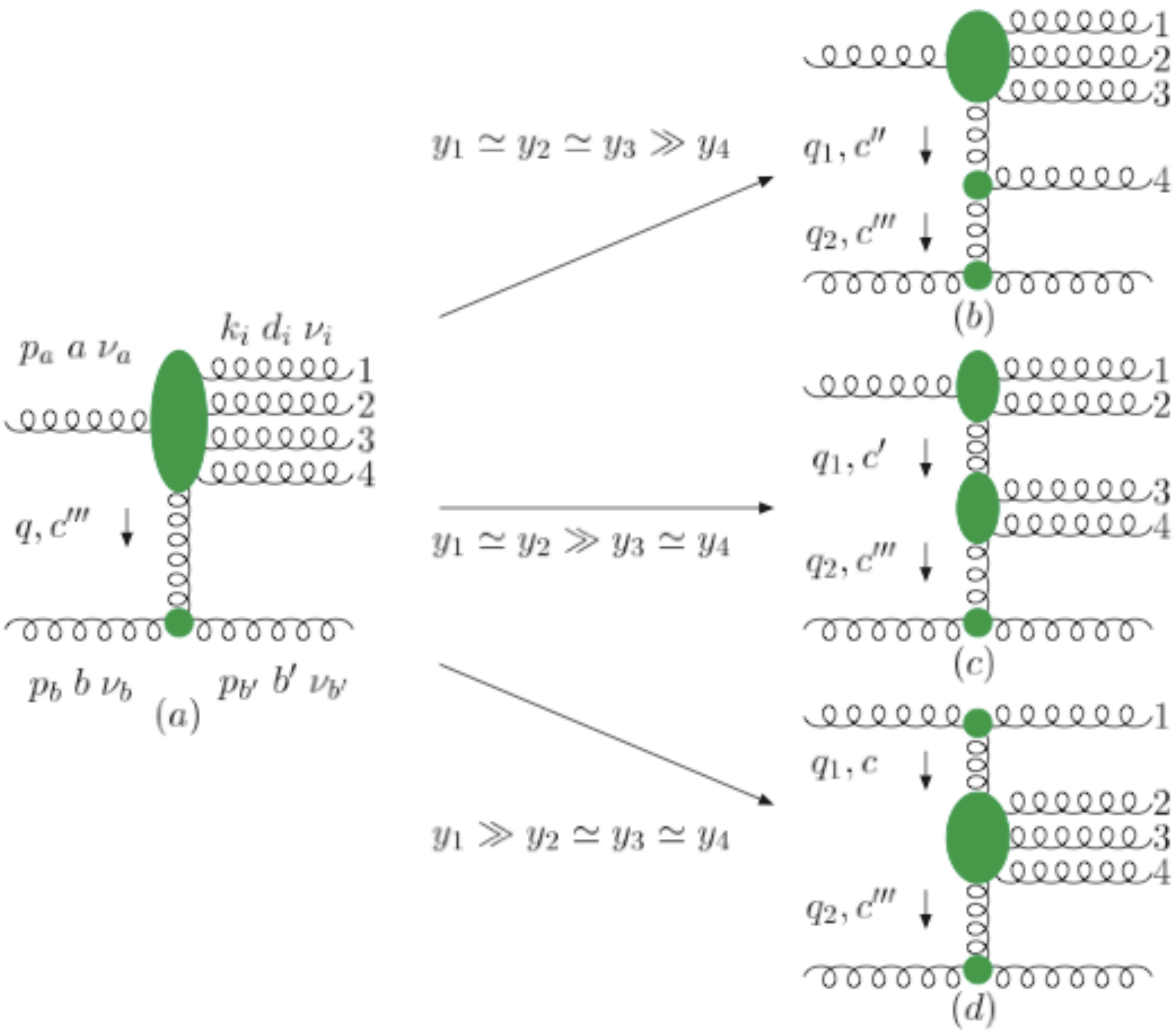
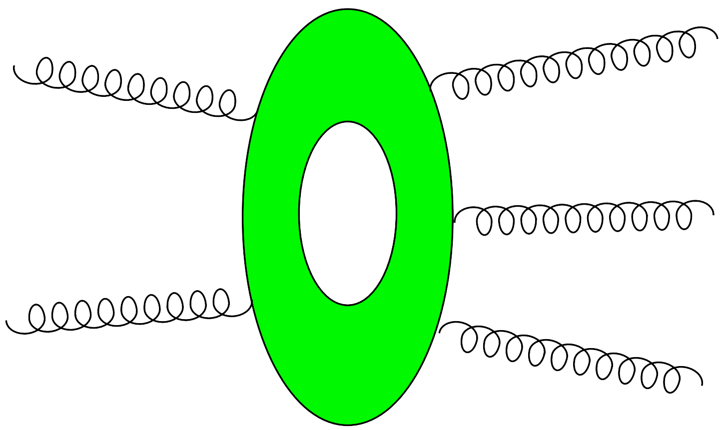
Frizzo Maltoni VDD 99

one may check several kinematic limits

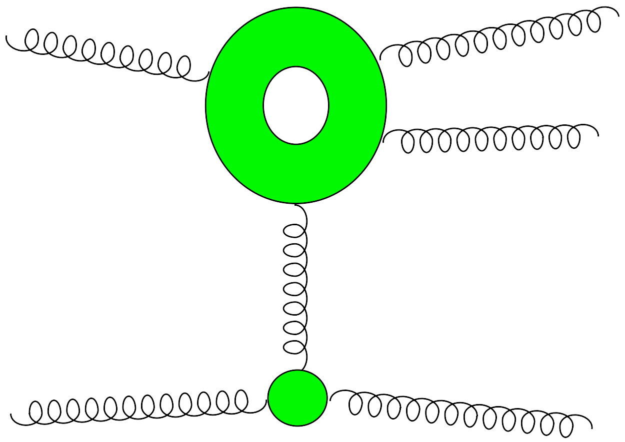


# Unknown 1-loop coefficient functions, which could be also computed ...



$y_1 \simeq y_2 \gg y_3$

$\supset$



boundary condition
to NNLL kernel

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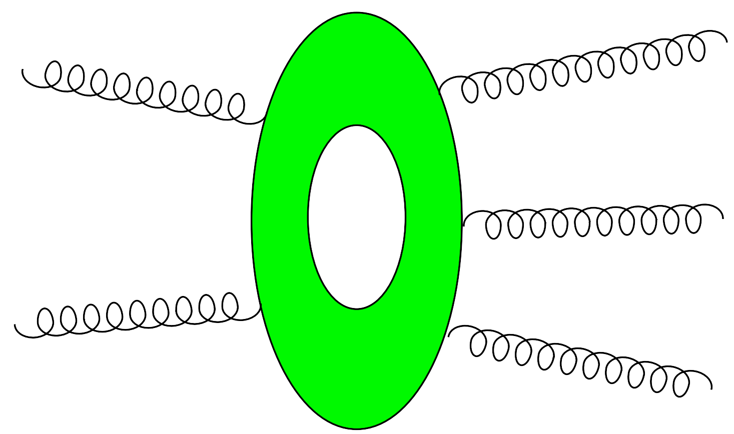

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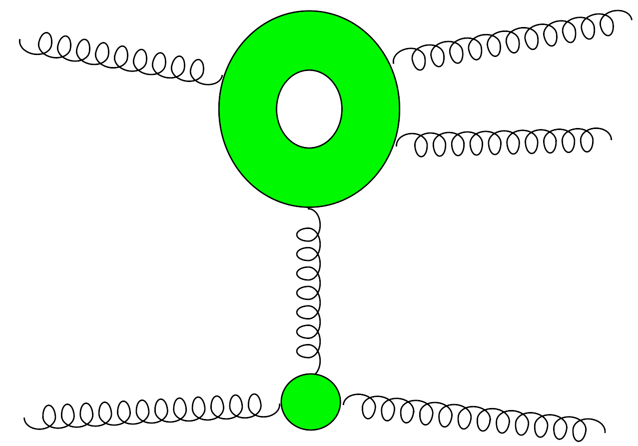

boundary condition
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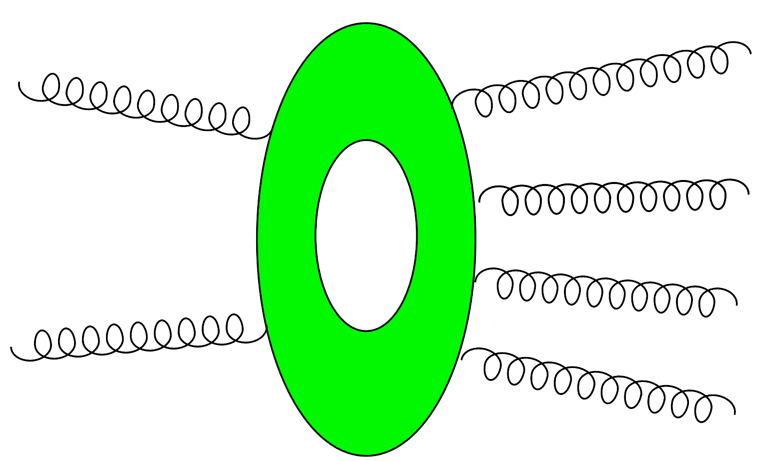

$y_1 \simeq y_2 \simeq y_3 \gg y_4$

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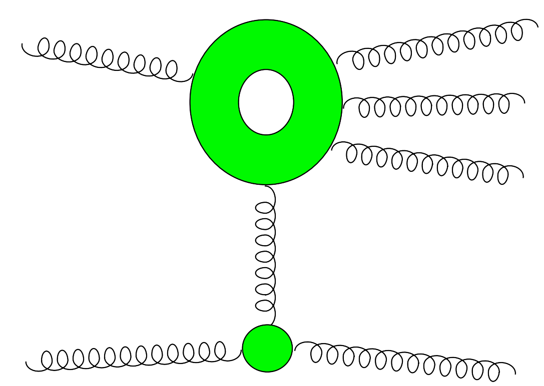

boundary condition
to NNNLL kernel

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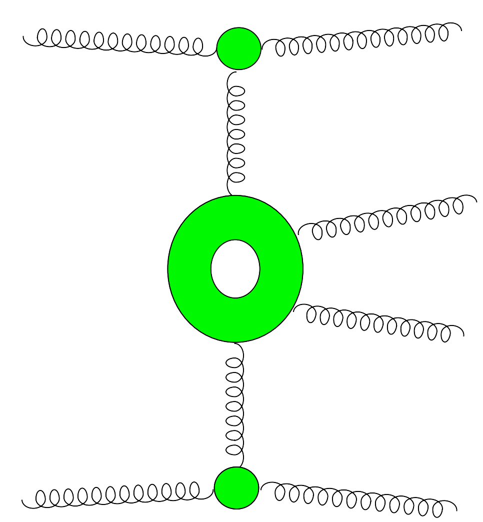
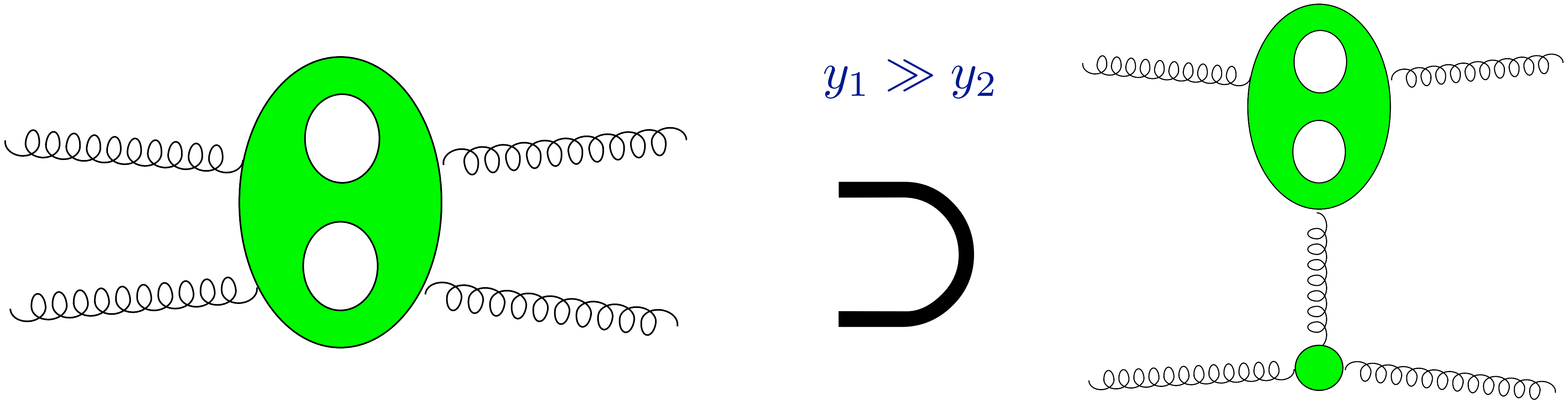


contributes to
NNLL kernel

In MSYM Bartels Lipatov Sabio-Vera 08

# as well as 2-loop coefficient functions ...




boundary condition
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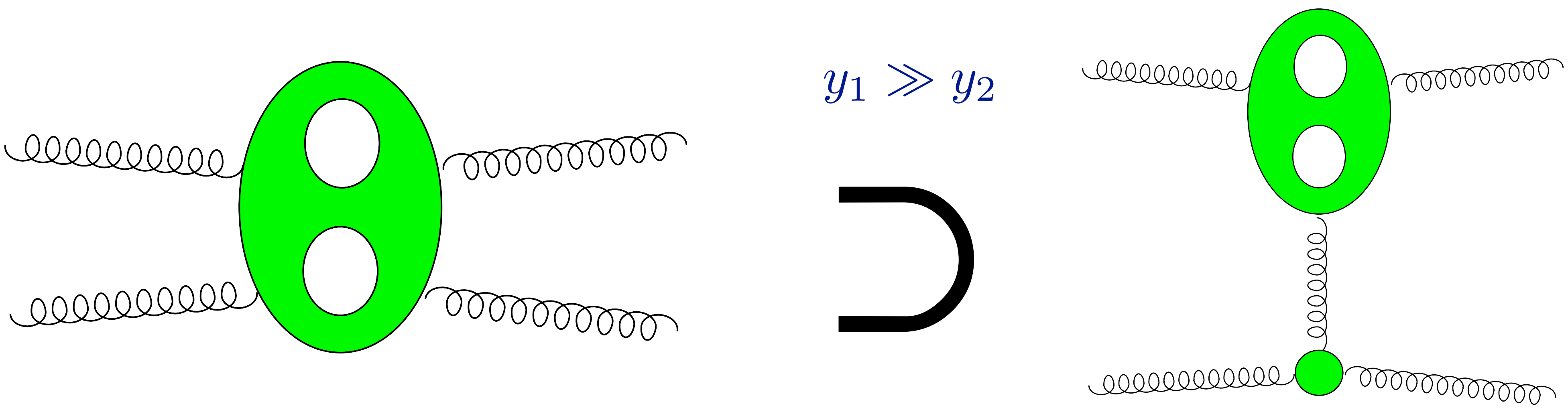


boundary condition
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The 1-loop and 2-loop coefficient functions I showed in the last two slides have never been computed in QCD. Why ? They are

- building blocks of BFKL kernels or of their boundaries, which, as of now, are unlikely to be built

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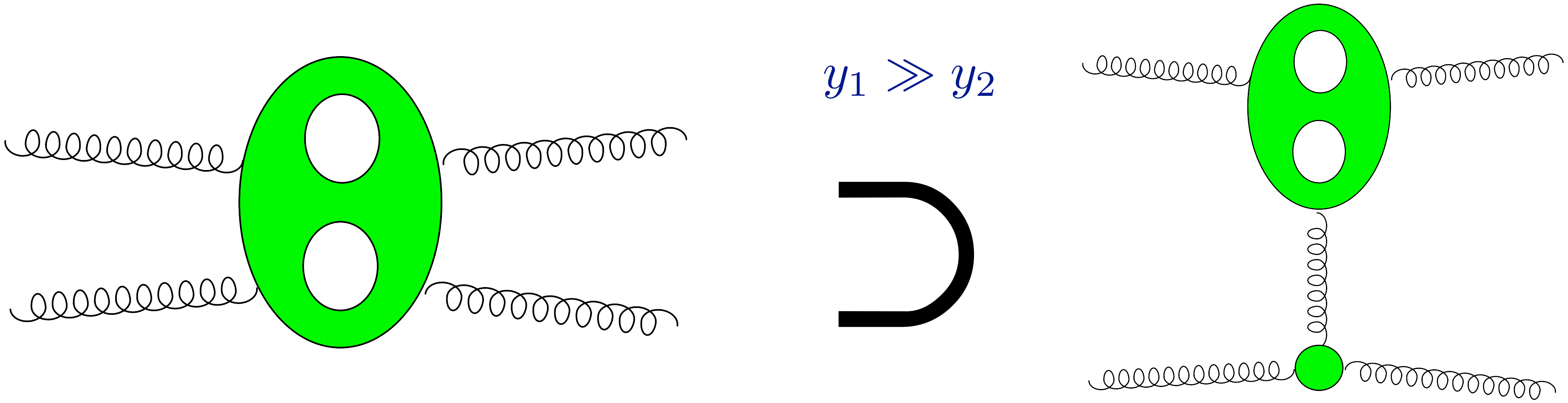


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- building blocks of BFKL kernels or of their boundaries, which, as of now, are unlikely to be built
- building blocks of n-point 1-loop or 2-loop amplitudes in particular kinematics, but in QCD we have no clue about the structure of n-point 1-loop or 2-loop amplitudes in arbitrary kinematics (except for 1-loop MHV configurations)

# N=4 Super Yang-Mills

Bern-Dixon-Smirnov computed the 2-loop 4-pt amplitude $M_4^{(2)}$ to $O(\varepsilon^2)$ and the 3-loop 4-pt amplitude $M_4^{(3)}$ to $O(\varepsilon^0)$.

Bern Dixon Smirnov 05

Those amplitudes can be used to test the high-energy factorisation of the 4-pt amplitude.

It is known that the factorisation formula for the QCD colour-dressed amplitude

$$M_4 = s\left[i\, g_s\, f^{aca'}\, C(p_a, p_{a'})\right] \frac{1}{t} \left[\left(\frac{-s}{-t}\right)^{\alpha(t)} + \left(\frac{s}{-t}\right)^{\alpha(t)}\right] \left[i\, g_s\, f^{bcb'}\, C(p_b, p_{b'})\right]$$

Fadin Lipatov 93

holds only up to NLL accuracy (which was fine for BFKL at NLL)

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Fadin Lipatov 93

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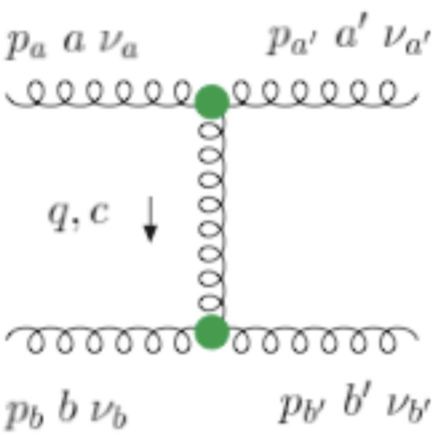
*Im* $M_4^{(1)}$ contains leading colour structures other than the f's Schmidt VDD 97

In the high-energy limit $m_4^{(0)}(-+-+) = -\, m_4^{(0)}(--++)$ at tree level

which are connected under $s \leftrightarrow u$ channel crossing.

Clearly, the coefficients of the colour-stripped amplitudes must be the same for the formula above to hold. At *n* loops, that occurs for the *n*-th log and for the *real part of the (n-1)*-th log: that suffices for BFKL at NLL natural to use a high-energy factorisation for the colour-stripped amplitude

$$m_4(-,-,+,+) \equiv m_4^s = s\,[g_s\,C(p_a,p_{a'})]\,\frac{1}{t}\left(\frac{-s}{-t}\right)^{\alpha(t)}\,[g_s\,C(p_b,p_{b'})]$$



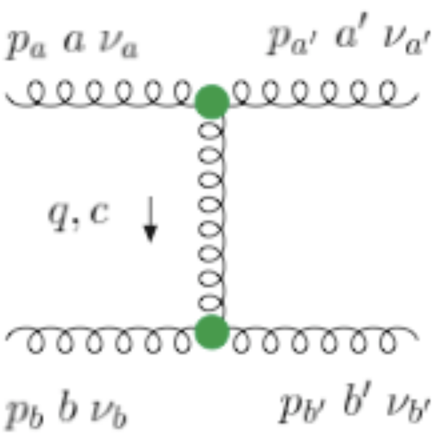

in the *s*-channel physical region

$$m_4(-,+,-,+) \equiv m_4^u = s\,[g_s\,C(p_a,p_{a'})]\,\frac{1}{t}\left(\frac{s}{-t}\right)^{\alpha(t)}\,[g_s\,C(p_b,p_{b'})]$$

in the *u*-channel physical region

The formulae above contain the same info: they are related by $s \leftrightarrow u$ channel crossing natural to use a high-energy factorisation for the colour-stripped amplitude

$$m_4(-,-,+,+) \equiv m_4^s = s\,[g_s\,C(p_a,p_{a'})]\,\frac{1}{t}\left(\frac{-s}{-t}\right)^{\alpha(t)}\,[g_s\,C(p_b,p_{b'})]$$




in the *s*-channel physical region

$$m_4(-,+,-,+) \equiv m_4^u = s\,[g_s\,C(p_a,p_{a'})]\,\frac{1}{t}\left(\frac{s}{-t}\right)^{\alpha(t)}\,[g_s\,C(p_b,p_{b'})]$$

in the *u*-channel physical region

The formulae above contain the same info: they are related by *s* ↔ *u* channel crossing

Using the high-energy limit of BDS's 2-loop 4-pt amplitude $M_4^{(2)}$ to O(ε²)
and 3-loop 4-pt amplitude $M_4^{(3)}$ to O(ε⁰),
one can check that the formulae above hold at 3-loop accuracy

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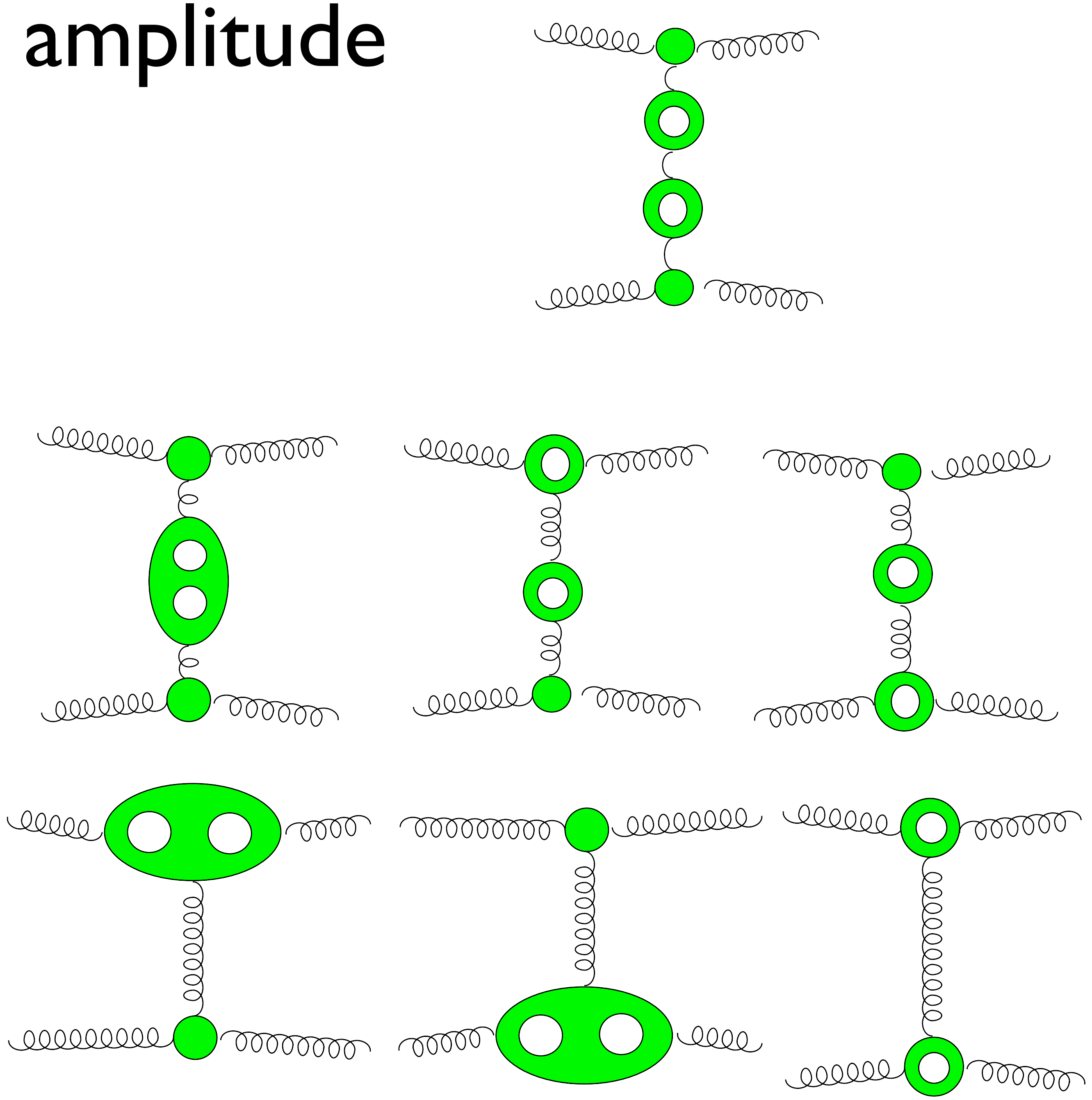
Instructive to implement the factorisation formulae with channel-dependent coefficient functions. If the test amplitudes are not in the ``right'' kinematics, the coefficient functions are indeed channel dependent → factorisation is broken

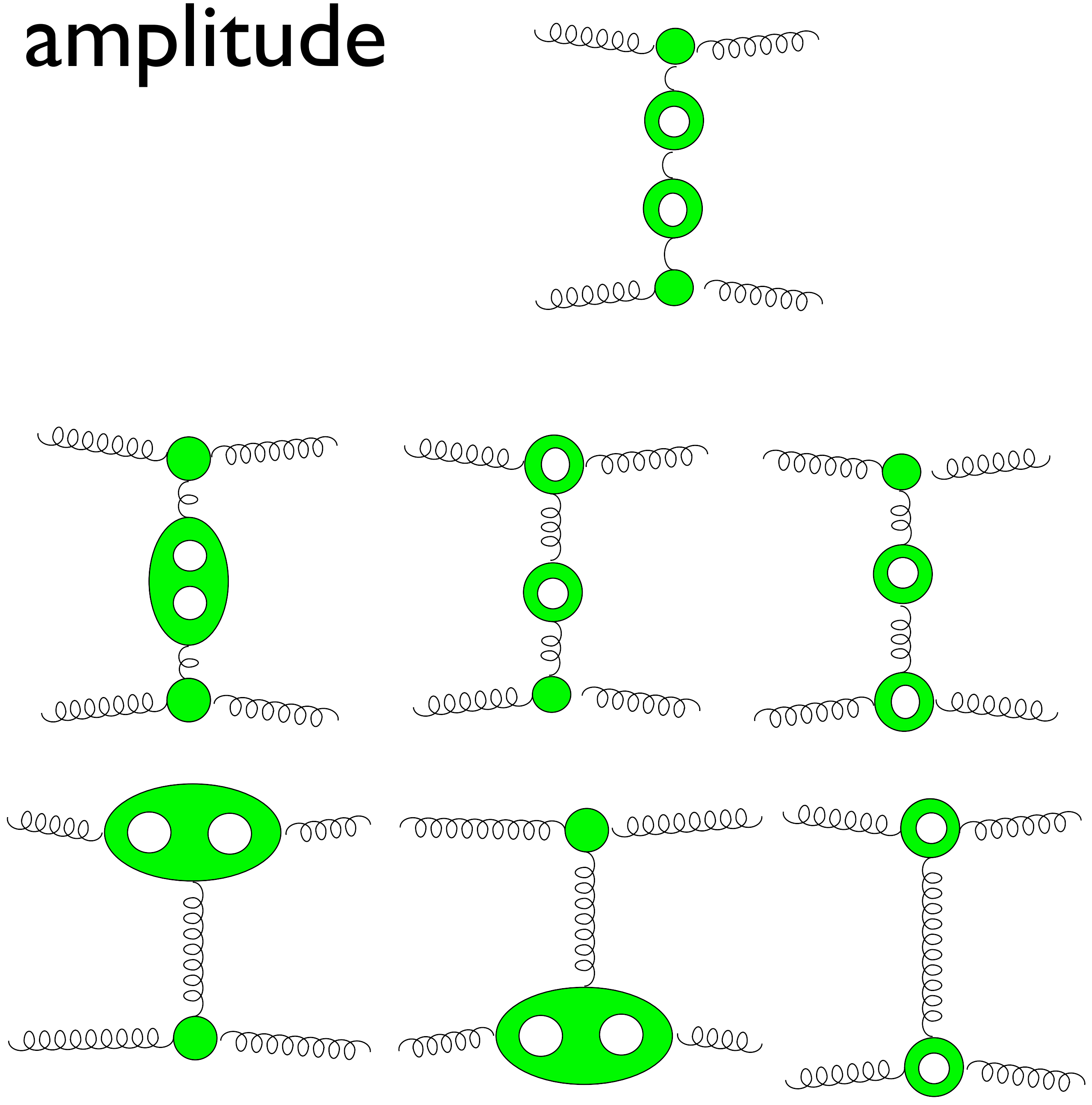
# Factorisation of the 2-loop amplitude

$$
\begin{aligned}
m_4^{u(2)} &= \frac{1}{2}\left(\alpha^{(1)}\right)^2 L^2 \\
&+ \left(\alpha^{(2)} + 2\,C^{(1)}\alpha^{(1)}\right)\, L \\
&+ 2\,C^{(2)} + \left(C^{(1)}\right)^2
\end{aligned}
$$

$$
L = \ln\left(\frac{s}{-t}\right)
$$



# Factorisation of the 2-loop amplitude



$$
\begin{aligned}
m_4^{u(2)} &= \frac{1}{2}\left(\alpha^{(1)}\right)^2 L^2 \\
&+ \left(\alpha^{(2)} + 2\,C^{(1)}\alpha^{(1)}\right) L \\
&+ 2\,C^{(2)} + \left(C^{(1)}\right)^2
\end{aligned}
$$

$$L = \ln\left(\frac{s}{-t}\right)$$

by direct calculation from
BDS's 2-loop 4-pt amplitude $M_4^{(2)}$ to $O(\varepsilon^2)$
we get 2-loop trajectory

$$\alpha^{(2)}_{MSYM} = -\frac{\pi^2}{3\epsilon} - 2\zeta_3 - \frac{4\pi^4}{45}\epsilon + (6\pi^2\zeta_3 + 82\zeta_5)\epsilon^2 + O(\epsilon^3)$$

2-loop coefficient function

$$C^{(2)}_{MSYM} = \frac{2}{\epsilon^4} - \frac{5\pi^2}{6}\frac{1}{\epsilon^2} - \frac{\zeta_3}{\epsilon} - \frac{11}{72}\pi^4 + \left(\frac{\pi^2}{6}\zeta_3 - 41\zeta_5\right)\epsilon - \left(\frac{95}{2}\zeta_3^2 + \frac{113\pi^6}{504}\right)\epsilon^2 + O(\epsilon^3)$$

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# BDS ansatz and high-energy factorisation

The BDS ansatz implies the 2-loop recursive formula
for the 2-loop 4-pt amplitude $m_4^{(2)}$ (rescaled by the tree amplitude)

$$m_4^{(2)}(\epsilon) = \frac{1}{2}\left[m_4^{(1)}(\epsilon)\right]^2 + \frac{2\,G^2(\epsilon)}{G(2\epsilon)} f^{(2)}(\epsilon)\, m_4^{(1)}(2\epsilon) - 2\,\zeta_2^2 + O(\epsilon)$$

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with $\quad f^{(2)}(\epsilon) = -\zeta_2 - \zeta_3\epsilon - \zeta_4\epsilon^2$

(we use a different normalisation from BDS) $\quad G(\epsilon) = \frac{e^{-\gamma\epsilon}\,\Gamma(1-2\epsilon)}{\Gamma(1+\epsilon)\,\Gamma^2(1-\epsilon)} = 1 + O(\epsilon^2)$

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Anastasiou Bern Dixon Kosower 03

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(we use a different normalisation from BDS) $\quad G(\epsilon) = \frac{e^{-\gamma\epsilon}\,\Gamma(1-2\epsilon)}{\Gamma(1+\epsilon)\,\Gamma^2(1-\epsilon)} = 1 + O(\epsilon^2)$

from the 2-loop recursive formula and high-energy factorisation, we get

$$C^{(2)}_{MSYM}(\epsilon) = \frac{1}{2}\left[C^{(1)}_{MSYM}(\epsilon)\right]^2 + \frac{2\,G^2(\epsilon)}{G(2\epsilon)} f^{(2)}(\epsilon)\, C^{(1)}_{MSYM}(2\epsilon) - \zeta_2^2 + O(\epsilon)$$

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one needs $C^{(1)}_{MSYM}$ through $O(\epsilon^2)$ but we know it to all orders of ε, in QCD

$$C^{(1)}_{MSYM} = \frac{\psi(1+\epsilon) - 2\psi(-\epsilon) + \psi(1)}{\epsilon}$$

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# BDS ansatz and 3-loop high-energy factorisation

from BDS's recursive formula for the 3-loop 4-point amplitude and high-energy factorisation, we get a recursive formula for the 3-loop coefficient function

$$C^{(3)}_{MSYM}(\epsilon) = -\frac{1}{3}\left[C^{(1)}_{MSYM}(\epsilon)\right]^3 + C^{(1)}_{MSYM}(\epsilon)\, C^{(2)}_{MSYM}(\epsilon) + \frac{4\,G^3(\epsilon)}{G(3\epsilon)} f^{(3)}(\epsilon)\, C^{(1)}_{MSYM}(3\epsilon) + 4\, Const^{(3)} + O(\epsilon)$$

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with $$f^{(3)}(\epsilon) = \frac{11}{2}\zeta_4 + (6\zeta_5 + 5\zeta_2\zeta_3)\epsilon + (c_1\zeta_6 + c_2\zeta_3^2)\epsilon^2$$

$$Const^{(3)} = \left(\frac{341}{216} + \frac{2}{9}c_1\right)\zeta_6 + \left(-\frac{17}{9} + \frac{2}{9}c_2\right)\zeta_3^2$$

one needs $C^{(2)}_{MSYM}$ through $O(\epsilon^2)$ and $C^{(1)}_{MSYM}$ through $O(\epsilon^4)$

# Conclusions

what's next ?
once the 2-loop 5-point amplitude
in the (quasi)-multi-Regge kinematics is known,
we can derive the corresponding coefficient functions
... work in progress

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A bootstrap approach:
once we know the coefficient functions from
the 2-loop 4-point and 5-point amplitudes,
we can use them to build 2-loop amplitudes
with 6 or more points, in the multi-Regge and
quasi-multi-Regge kinematics, and thus obtain
(hopefully useful) info on the analytic form of
2-loop amplitudes with 6 or more points in
arbitrary kinematics