A holographic interpretation of the Regge region

Luis Alvarez-Gaume Zurich ETH July 5th 2008

Work in collaboration with
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General philosophy

One of the most important lessons we have learned from the Maldacena conjecture is that the QCD string, is the fundamental string in some higher dimensional geometry. Accumulated evidence points to the relevance of BH's in the holographic description

There are two approximations to YM theories where the string picture appears naturally. One is the 't Hooft large-N limit. For strong 't Hooft coupling, we have a geometric description, at least with enough supersymmetry

The Regge limit, historically at the origin of string theory, with finite g and N, but with very large Log s. In this kinematic regime, resummation of diagrams in terms of (g2 Log s) leads to a stringy behavior of the form $s^{\alpha(t)}$
Is there a holographic interpretation of this behavior?

Introductory remarks

Gravitational collapse is one of the most fascinating subjects in TH Physics

There are important problems at the classical level...

and even more tantalizing ones at the quantum level...

Important progress has taken place
over the last two decades

Classical	"Quantum"
Cosmic censorship	BH state counting
Numerical relativity	Zoology of black objects
Critical phenomena	AdS/CFT a revolutionary
BHW Entropy formula	way to look at the problem
No-hair theorem revisited	Gravity versus YM Theory

Scaling phenomena

Gravity ⟷ Yang-Mills

Holography

Our work is inspired by the Maldacena conjecture

and we believe that it should be extended beyond supersymmmetry

So far most scenarios explored have been static, i.e. thermal HP...

We want to find dynamical phenomena that could be related and also observable:

Weakly coupled gauge theories vs strong curvature gravity

...continued

We compare universal properties of BH formation in the scaling region

with

The scaling properties of the Regge region in YM, a weakly coupled, but non-perturbative regime. The hard pomeron world

Scaling in gravitational collapse

Studying rigorously gravitational collapse for a m=0 field coupled to gravity

to answer questions of black hole formation from regular initial conditions,

and also the possible appearance of naked singularities, Christodoulou asked:

Is it possible to create black holes with

arbitrarily small mass?

The answer is yes: Type I,II collapse

Critical behavior in phase space

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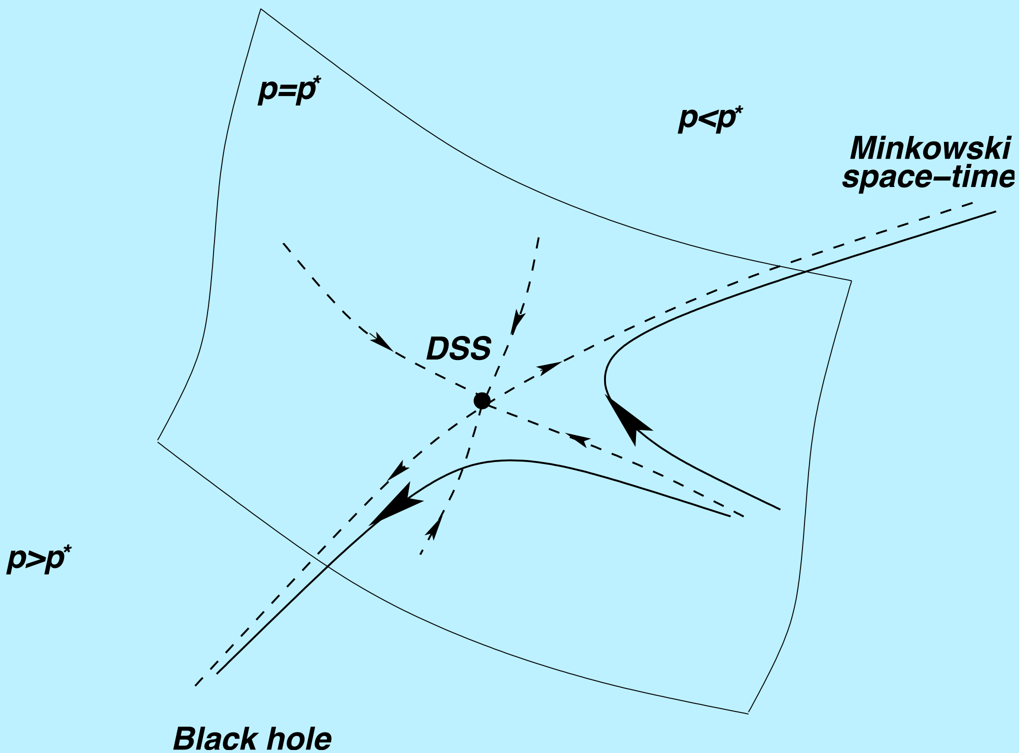

Choptuik's (93) showed the existence of a co-dimension one critical surface.

For generic one parameter families of initial data, parameterized by p, there is a critical value p* where it crosses the critical surface.

There are two possible large time evolutions, or fixed points:

A BH forms with arbitrarily small mass

Or the system bounces and it is radiated away to infinity leaving behind M4

The critical solution has an unstable mode, or relevant direction.

The eigenvalue of the relevant direction leads to the BH critical exponent.

Basic results

For the spherical collapse of the massless scalar field, the metric takes the form:

$$ds^2 = -\alpha^2(t,r)dt^2 + a^2(t,r)dr^2 + r^2 d\Omega^2_{\mathrm{d}-2}$$

By looking at one-parameter families of initial conditions, Choptuik found the existence of a critical solution, there are two basic properties:

The critical solution is independent of the initial conditions. On the supercritical side, the size of the small BH satisfies a universal scaling law. The critical solution exhibits DSS:

$$r_{BH} \sim (p-p^*)^\gamma \quad ; \quad Z_*(e^{n\Delta}\,t, e^{n\Delta}\,r) = Z_*(t,r)$$

Choptuik vs Liapounov

The numerical values obtained depend only on the type of matter considered, they are "pure" numbers. They do not depend on initial conditions. The critical solution is characterized by having a single unstable direction. Hence computing the Choptuik exponent is related to computing the Liapounov exponent of the small perturbations around the critical solution

$$Z_p(\tau,\zeta) \approx Z_*(\tau,\zeta) + \sum_{k=1}^{\infty} C_k(p) e^{\lambda_k \tau} \delta_k Z(\tau,\zeta)$$

D	Δ	γ
4	$3.37 \pm 2\%$	$0.372 \pm 1\%$
5	$3.19 \pm 2\%$	$0.408 \pm 2\%$
6	$3.01 \pm 2\%$	$0.422 \pm 2\%$
7	$2.83 \pm 2\%$	$0.429 \pm 2\%$
8	$2.70 \pm 2\%$	$0.436 \pm 2\%$
9	$2.61 \pm 2\%$	$0.442 \pm 2\%$
10	$2.55 \pm 3\%$	$0.447 \pm 3\%$
11	$2.51 \pm 3\%$	$0.44 \pm 3\%$

$$\gamma = -\frac{1}{\lambda_1}$$

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Perfect fluid collapse

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In the relevant scaling limit in YM, there is no echo parameter.

We want a similar symmetry in gravity. This is achieved by studying the collapse of perfect fluids.

The critical solution will have CSS rather than DSS. A region of the space time before the singularity forms has homothety, i.e. a conformal Killing vector of weight 2.

We choose comoving coordinates to describe the spherical collapse of the fluid. The equations are simpler.

Cahill-Taub, Bicknell-Henriksen, Coleman-Evans, Hara-Koike-Adachi, Harada-Maeda. We follow and complete these authors in any d

$$ds^2 = -\alpha(t,r)^2 dt^2 + a(t,r)^2 dr^2 + R(t,r)^2 d\Omega_{d-2}^2$$

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p\, g_{\mu\nu}$$

$$p = k\,\rho, \qquad 0 \le k \le 1$$

Equations of motion

$$
\begin{aligned}
2m_{,r} &= \frac{16\pi}{d-2}\,\rho R_{,r} R^{d-2}, \\
2m_{,t} &= -\frac{16\pi}{d-2}\,p R_{,t} R^{d-2}, \\
2G_N m &= R^{d-3}\left(1+\frac{R_{,t}^2}{\alpha^2}-\frac{R_{,r}^2}{a^2}\right), \\
\frac{\alpha_{,r}}{\alpha} &= -\frac{p_{,r}}{\rho+p}, \\
\frac{a_{,t}}{a} &= -\frac{\rho_{,t}}{\rho+p}-(d-2)\frac{R_{,t}}{R},
\end{aligned}
$$

It is easy to understand physically each of these equations

CSS conditions

$$\tau = -\log(-t), \qquad z = -\frac{r}{t}$$

$$\begin{aligned} \eta(\tau, z) &= 8\pi r^2 \rho(t, r), \\ S(\tau, z) &= \frac{R(t, r)}{r}, \\ m(t, r) &= r^{d-3} M(t, r), \end{aligned}$$

$$y = (d-2)(d-3)\frac{M}{\eta S^{d-1}}$$

$$\alpha = c_\alpha(\tau)\left(\frac{z^2}{\eta}\right)^{\frac{k}{k+1}}, \qquad a = \eta^{-\frac{1}{k+1}} S^{2-d} \qquad V_z = -\frac{a\, z}{\alpha}$$

$$\begin{aligned} \frac{M'}{M} + (d-3) &= \frac{d-3}{y}\left(1 + \frac{S'}{S}\right) \\ \frac{\dot{M}}{M} + \frac{M'}{M} &= -\frac{(d-3)k}{y}\left(\frac{S'}{S} + \frac{S'}{S}\right) \\ a^2 S^{-2}\left(\frac{2M}{S^{d-3}} - 1\right) &= V_z^2\left(\frac{\dot{S}}{S} - \frac{S'}{S}\right)^2 - \left(1 + \frac{S'}{S}\right)^2 \end{aligned}$$

Regularity conditions

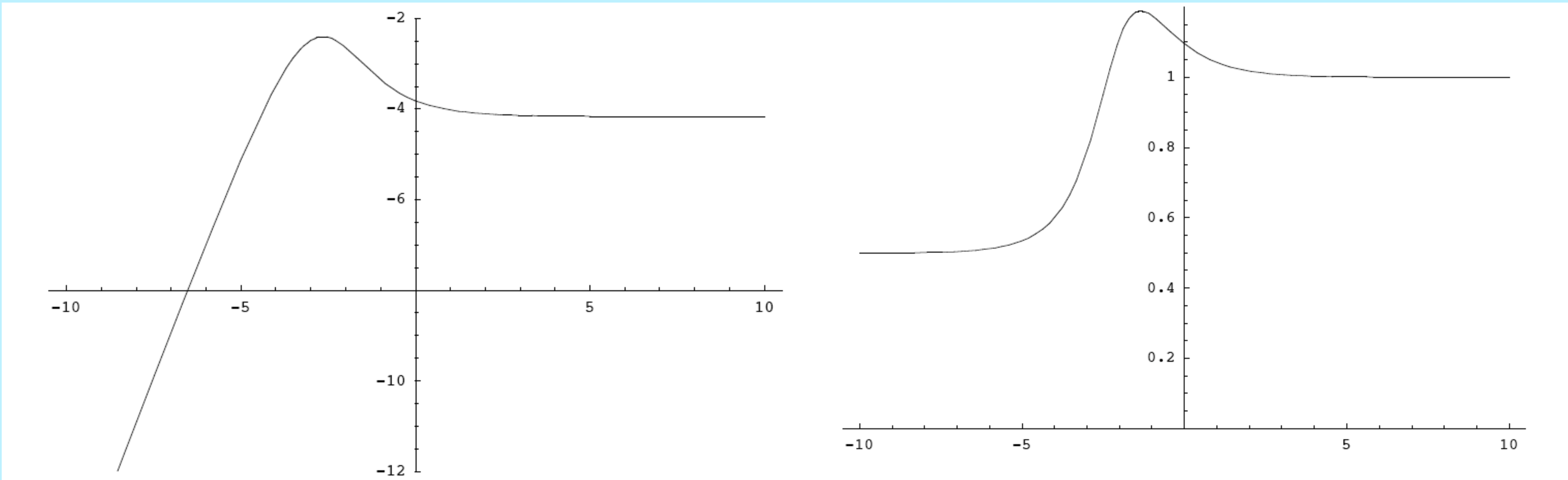
$$\frac{d\log M}{d\log z} = \frac{(d-3)k}{k+1}\left(\frac{1}{y}-1\right)$$
$$\frac{d\log S}{d\log z} = \frac{1}{k+1}(y-1)$$
$$\frac{d\log \eta}{d\log z} = \frac{1}{V_z^2-k}\left[\frac{(1+k)^2}{d-2}\eta^{\frac{k-1}{k+1}}S^{4-2d}-(d-2)(y-1)V_z^2-2k\right]$$

$$y(0^+) = \frac{d-3}{d-1}$$

$$M(z) \simeq \frac{(2D)^{\frac{k}{k+1}}}{(d-2)}\left[\frac{k+1}{(d-1)k+d-3}\right]z^{\frac{2k}{k+1}}$$
$$S(z) \simeq \left[\frac{(2D)^{\frac{1}{k+1}}}{k+1}\left(k+\frac{d-3}{d-1}\right)\right]^{\frac{1}{1-d}} z^{-\frac{2}{(d-1)(k+1)}}$$

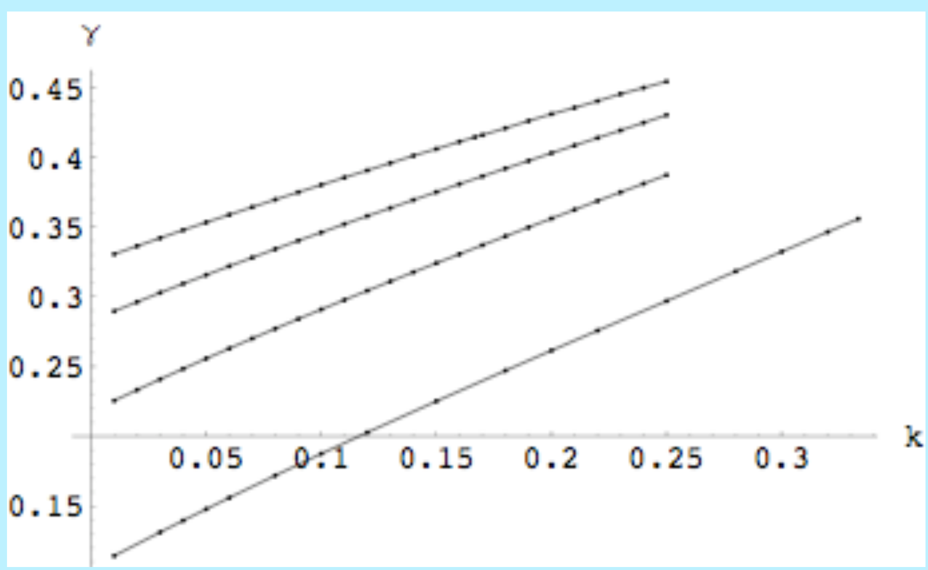
Analyticity at the origin and sonic surface determines a discrete set of D's only functions of k,d. For each such D we can uniquely determine the Choptuik exponent by analyzing the perturbations. Here we have gone beyond what is in the literature, where a crucial equation is missing

Sample numerics



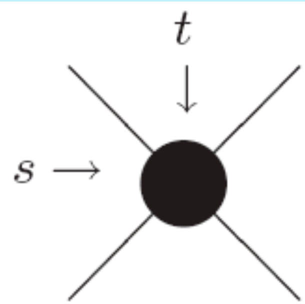
Plot of the critical solution for k=.05, d=5, D=80.62

In the analysis of the perturbations, and the computation of exponents, it is important also to take into account analyticity at the sonic point



BFKL the Regge limit of YM

BFKL is an equation which describes the high-energy limit of weakly coupled YM



$s >> |t|\,, Q^2$

$\alpha_s(Q^2) << 1$

Large $\ln s$ compensate small α_s: $\alpha_s \ln s \sim 1$ [Balitsky–Fadin–Kuraev–Lipatov]

$$\mathcal{A}(s,t) \sim s^{\alpha(t)} \quad ; \quad s >> 1 \qquad \sigma^{\text{total}} \sim s^{\alpha(0)-1}$$

$$\alpha(0) = 1 + (4 \log 2)\, \alpha_s + \mathrm{O}[\alpha_s^2]$$

Unitarity violation, Froissart-Martin bound

Reggeized particles

The notion of reggeization is crucial in the soft pomerons, and it is fundamenal in the BFKL and BK equations

A particle of spin j and mass m reggeized if when exchanged in the t-channel, the amplitude behaves as

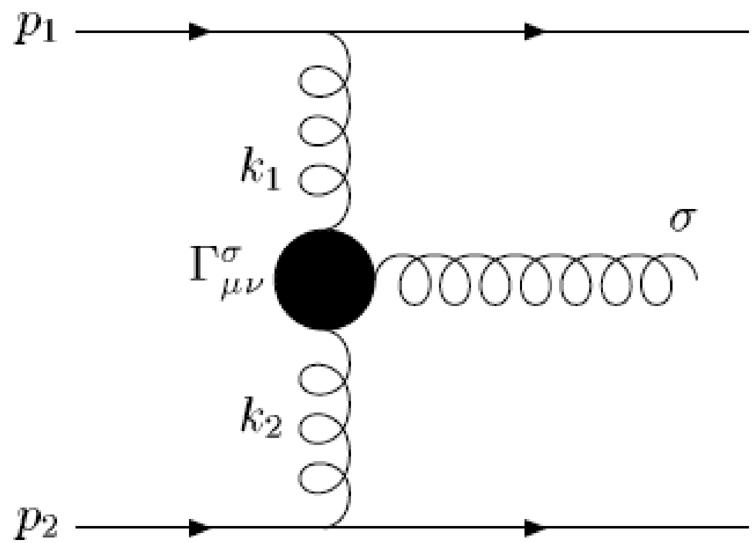
where in the exponent we have the corresponding Regge trajectory.

$$s^{\alpha(t)}$$

A substantial amount of work shows that the gluon reggeizes in the regge limit

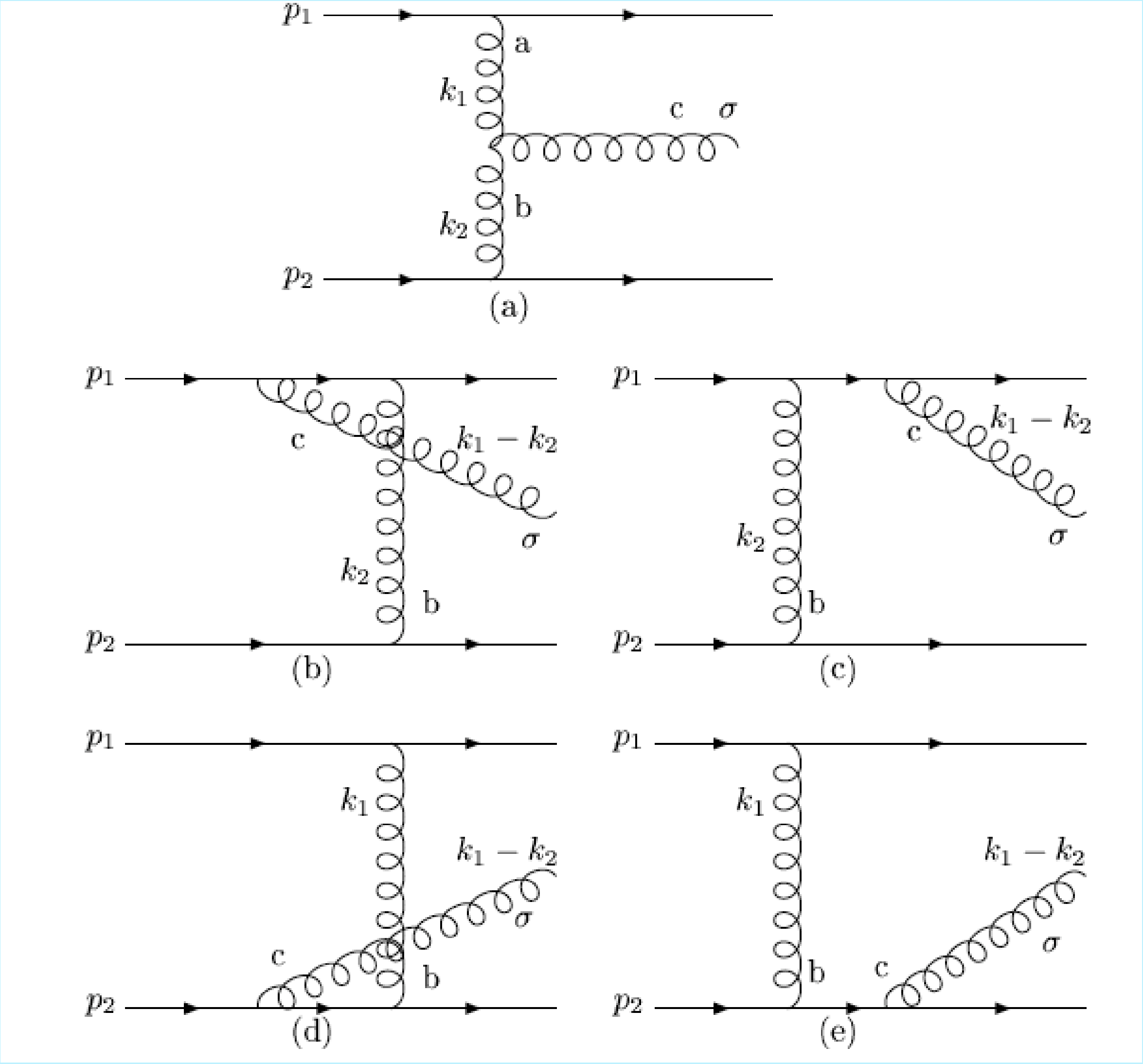
Use of the cutting rules is crucial. In the next few pages we assume that we are computing the absorptive part, and later discuss the real part as well

$$k_1^+ \gg k_2^+,\ k_2^- \gg k_1^-$$

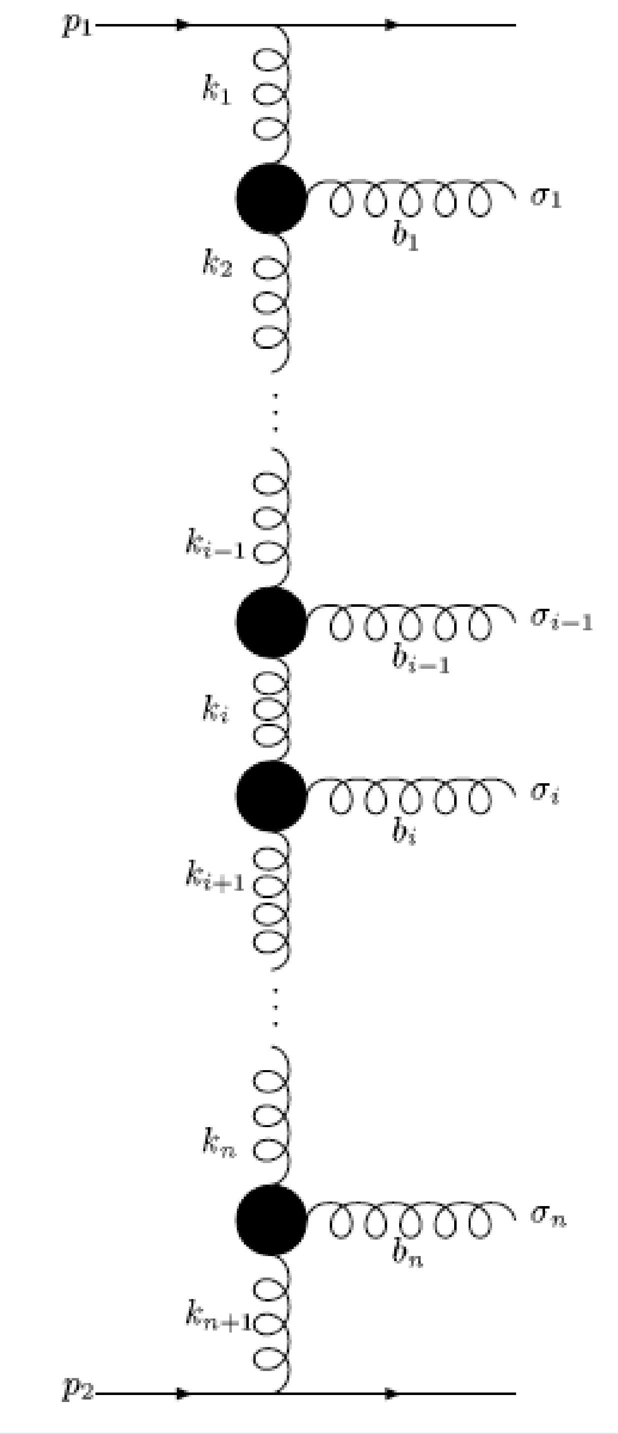


$$\Gamma^\sigma_{+-}(k_1,k_2) \;=\; 2gf^{abc}\left(k_1^+ + \frac{2\mathbf{k_1}^2}{k_2^-}, k_2^- + \frac{2\mathbf{k_2}^2}{k_1^+}, -(\mathbf{k_1}+\mathbf{k_2})\right)$$

Graphs contributing to the effective vertex



Effective ladder graphs

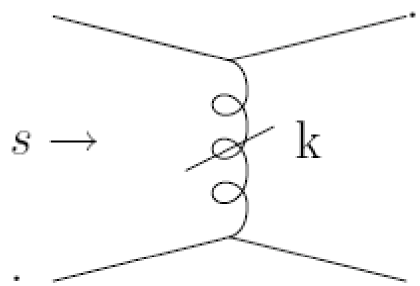


Generalizes to all order in the leading log approximation as is the case in simpler examples keeping the original vertices

Using large amounts of algebra, one can prove reggeization of the gluon. Each vertex is replaced by the effective vertex.

It can be shown that effective planarity is recovered, but now the effective N is related to the original N, and log s

Reggeized gluon

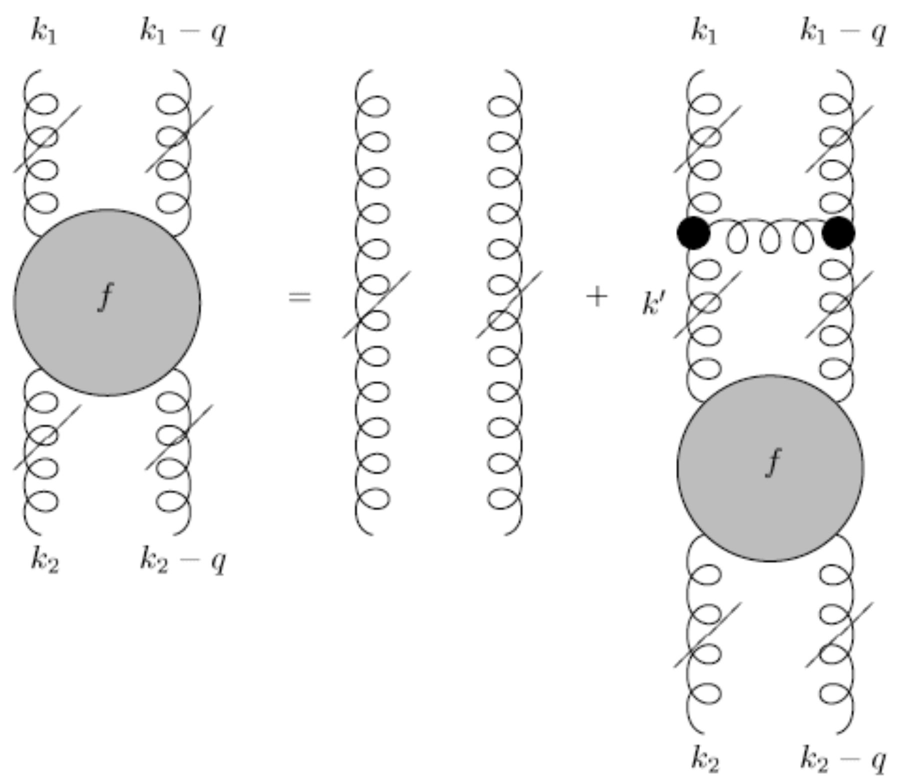


$$\frac{1}{\mathbf{k}^2}\left(\frac{s}{\mathbf{k}^2}\right)^{\varepsilon_G(\mathbf{k}^2)}$$

$$\epsilon_G(\mathbf{q}^2) = -\frac{\alpha_s C_A}{4\pi^2}\int d^2\mathbf{k}\,\frac{\mathbf{q}^2}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}$$

Some details, and RG

Like a Bethe-Salpeter equation at leading log



$$f(\sqrt{s},\mathbf{k_1},\mathbf{k_2}) = \delta^2(\mathbf{k_1}-\mathbf{k_2}) - \frac{\alpha C_A}{2\pi^2}\int d^2\mathbf{k}'\int^{\sqrt{s}} dk^+ f(k^+,\mathbf{k}',\mathbf{k_2})$$

$$\times \left(\frac{k_1^+}{k^{+\prime}}\right)^{2\varepsilon_G(\mathbf{k}'^2)} \frac{\Gamma^{\sigma}_{+-}(k_1,k')\Gamma^{\sigma}_{+-}(k_1,k')}{\mathbf{k}'^4}$$

This is similar to integrating out the fast, longitudinal degrees of freedom and working with the effective transverse hamiltonian.

This hamiltonian exhibits scale (SL(2,C)) invariance

Eigenfunctions and eigenvalues

Eigenfunctions:, $\phi_{n,\nu}(\mathbf{k}) = (k^2)^{-1/2+i\nu} e^{in\theta}$

Eigenvalues: $\frac{\alpha_s C_A}{\pi} \chi_n(\nu)$

$$\chi(\nu) = 2\Psi(1) - \Psi\left(\frac{(n+1)}{2} + i\nu\right) - \Psi\left(\frac{(n+1)}{2} - i\nu\right)$$

General solution $(\mathbf{k} = (k, \theta))$

$$\tilde{f}(\omega, \mathbf{k_1}, \mathbf{k_2}) = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{d\nu}{2\pi^2 \mathbf{k_1} \mathbf{k_2}} \left(\frac{\mathbf{k_1}^2}{\mathbf{k_2}^2}\right)^{i\nu} \frac{e^{in(\theta_1 - \theta_2)}}{\omega - \overline{\alpha_s} \chi_n(\nu)}$$

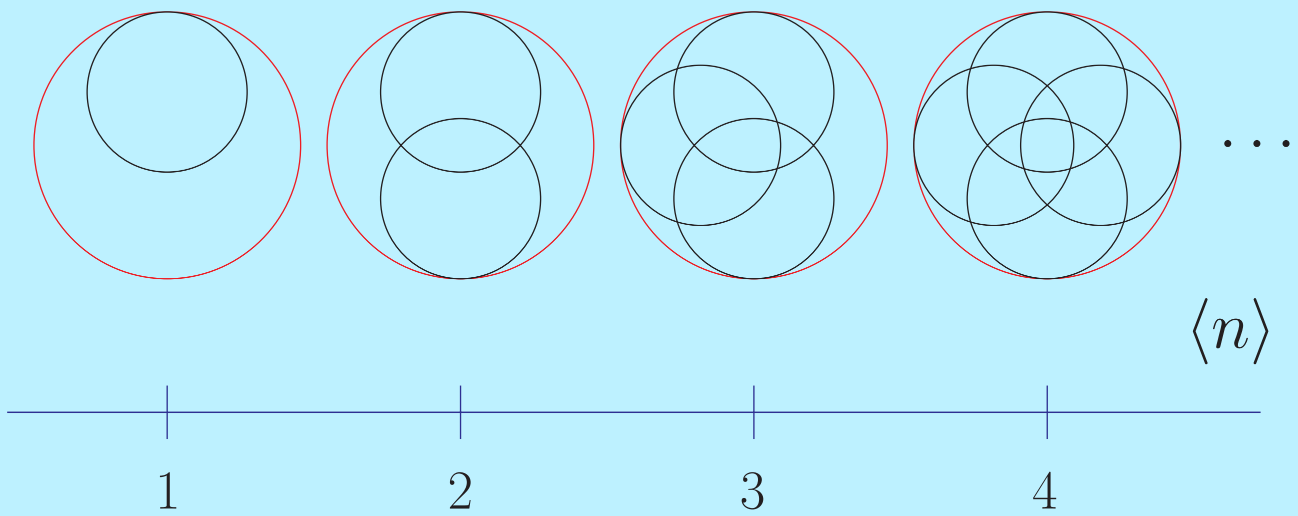
where

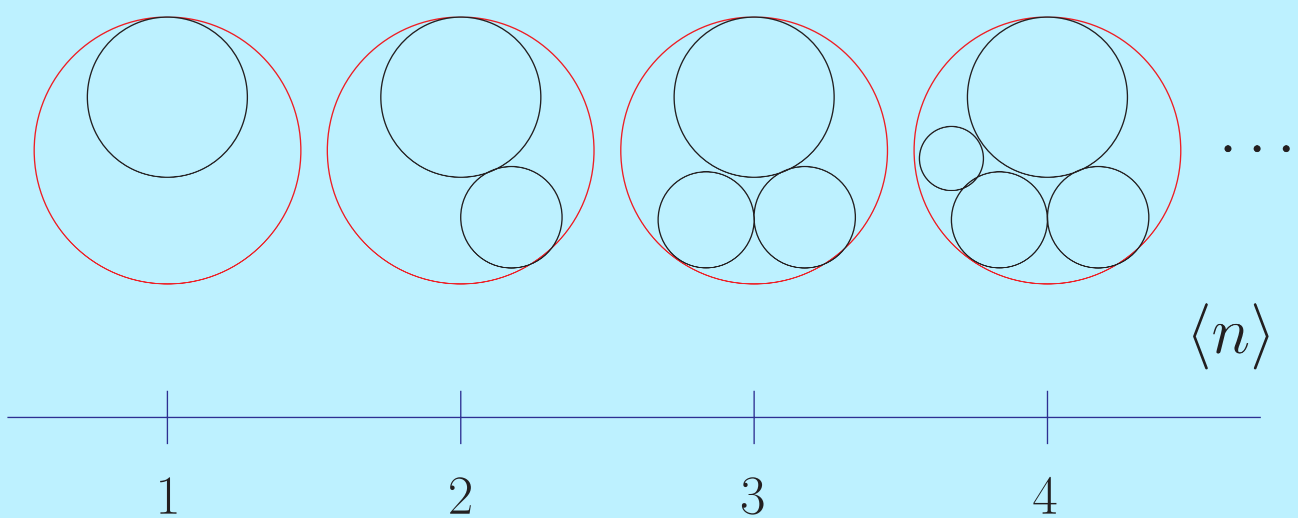
$$\overline{\alpha_s} \equiv \frac{\alpha_s C_A}{\pi}$$

Therefore BFKL is valid up to a “saturation scale'' after which nonlinear effects from overlapping wave function of gluons and partons cannot be neglected.

The BFKL can be modified to introduce nonlinear effects to restore unitarity. These lead to the saturation phenomena, easier to explain in terms of pictures. This is the BK behavior of the gluon distribution function

A pictorial description





Dilute to dense transition, with a fractal dim equal to the BFKL exponent

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Saturation in QCD

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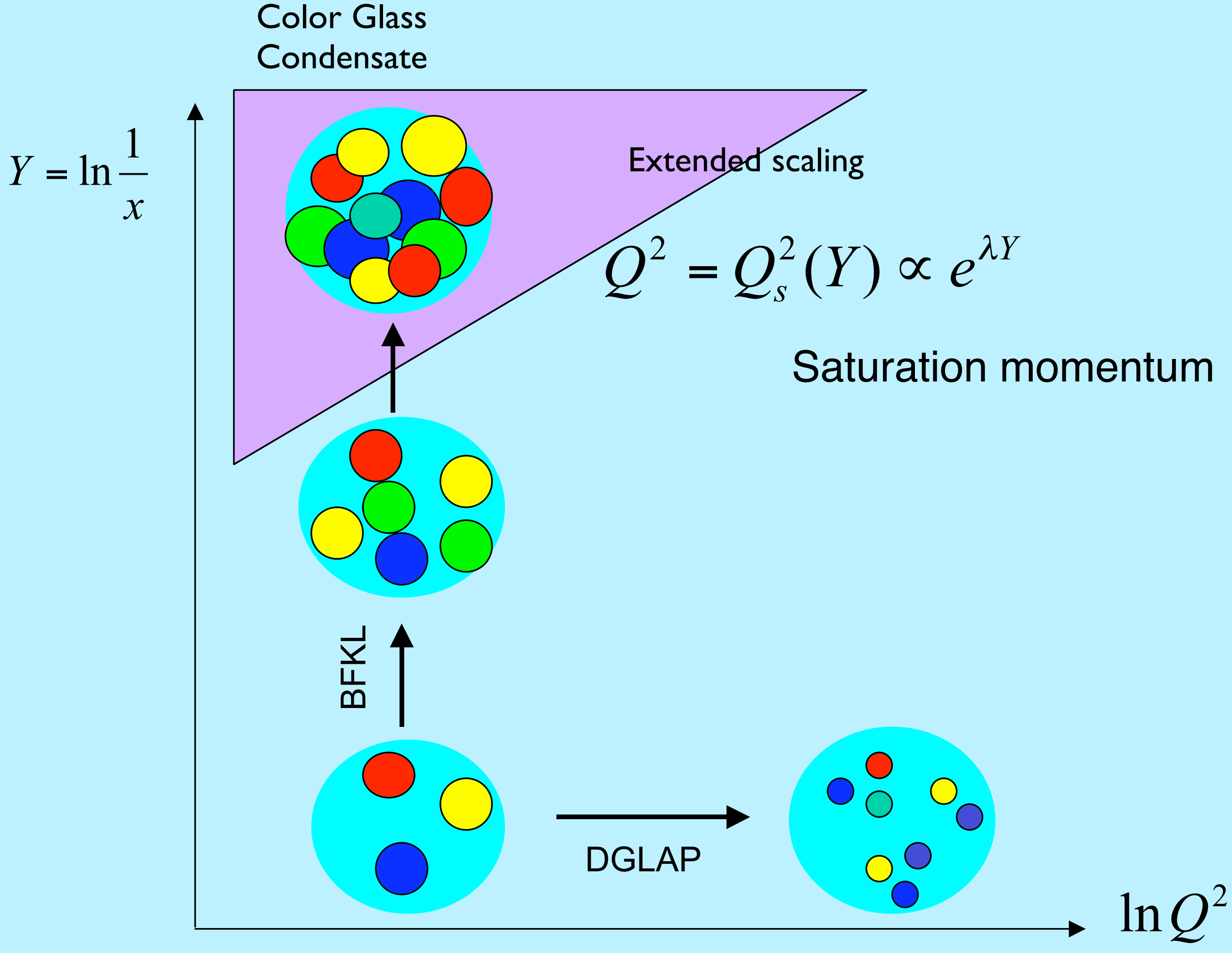
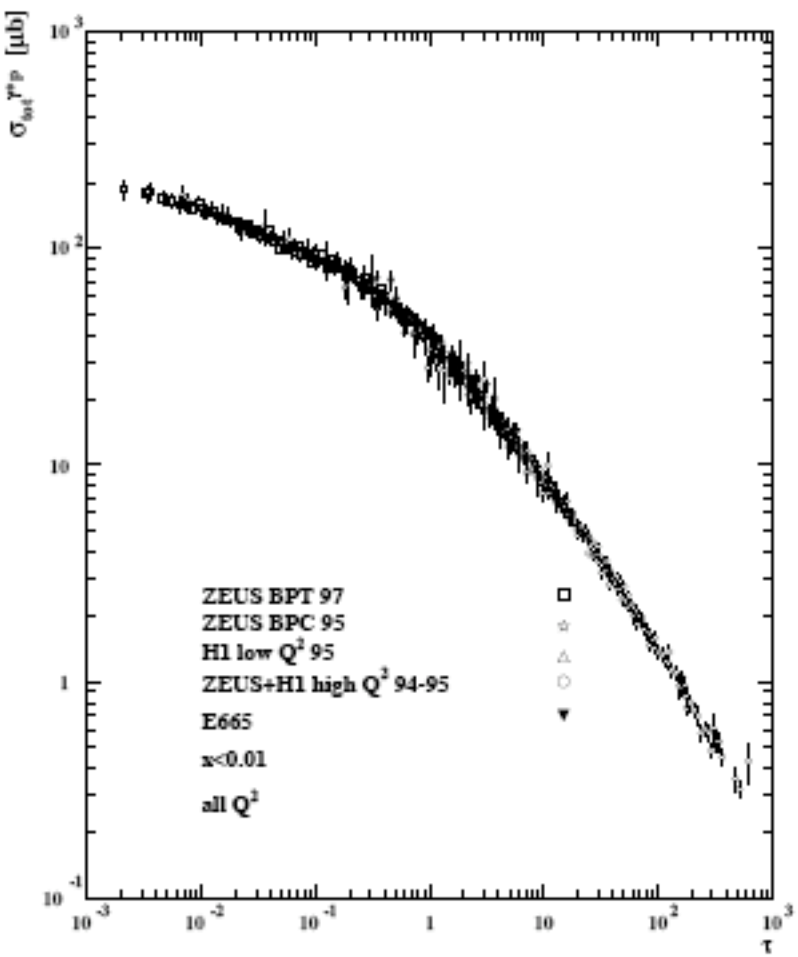


Geometric scaling

$\sigma^{\text{tot}}_{|\gamma^* \text{ P}}(Y,Q) = \sigma^{\text{tot}}_{|\gamma^* \text{ P}}(\tau) \ ; \ \tau = \frac{Q^2}{Q_s^2(Y)} = Q^2 x^{\lambda_s}$ HERA data for $\sigma_{\gamma^* p}$with $x < 0.01$versus τ



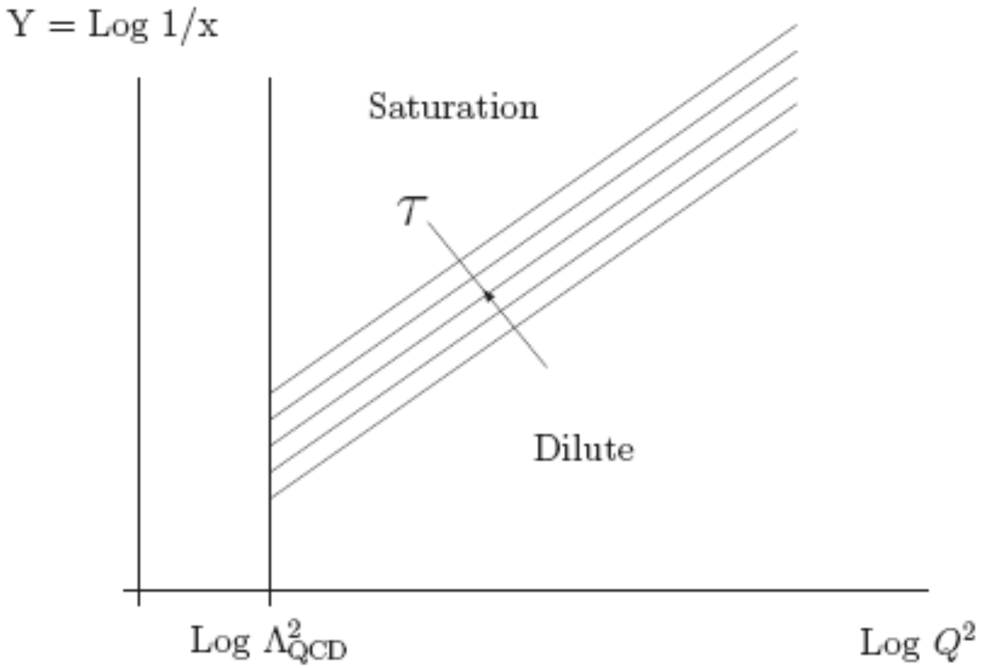


Fig. 5: Continuous self–similarity on the $(Y, \log Q^2)$ plane.

Some more speculation

BFKL to BK is like going from single particle Hilbert space to the Fock space

Klein paradox in RQM

Horowitz-Pochinski transition for small BHs

The unitarization in the Regge region may help, through holography to understand what happens at least with small BHs

$$\gamma_{\rm cr} = .6275 \qquad \lambda_{\rm cr} = \chi'(\gamma_{\rm cr})/2 = 2.44...$$

Rough dictionary

4D YM	5D Gravity
Weakly coupled	Strongly coupled
Large s	Large velocities
Geometric Scaling	CSS perfect fluid
$(\ \alpha_s N_c Y\ ,\ \log\ Q^2\ )$	(t, r)
Scaling symmetry of BFKL/BK	Conformal perfect fluid k=1/4

Final comments

Effective planarity at leading order in the Regge or multi-regge region

$$N_{eff} \sim \frac{\alpha N}{4\pi} \log s$$

Robustness based on the fact that we compute critical exponents, replaces BPS and holomorphy Not as robust but reasonably strong

The corresponding exponents of geometric scaling and CSS are reasonably close.

$$\lambda_{\mathrm{BFKL}} \approx 2.44 \approx \lambda_{\mathrm{Ch}}^{d=5,k=1/4}$$

The LHC will begin exploring systematically the Regge region, the exponents will be measured accurately

The fate of the BH singularity. SHP transitions one-to-multi string transition

$$\lambda_{\mathrm{SBH}} = 2.58$$

More work needed to sharpen our ideas.

Speculation galore...

Our computation relied on spherical symmetry

There are better ways to look at holographic representations of Regge collisions, like the collision of plane gravitational waves (Abraham and Evans)

In the BK equation we have two fixed points, one at low density is described by the BFKL equation with an unstable direction

At high densities we have a stable fixed point that restores unitarity and energy conservation. This is the saturation region.

In the gravitational context we have also two fixed points:

The Choptuik critical solution with a single unstable direction

and the AdS BH that becomes static after all available energy has been absorbed.

There is a cross over between the two, where scaling should show up.
The analogy is irresistible

....Continued

Type IIB collapse

Small vs big AdS black holes

Horowitz-Hubeny Solution

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Thank you